# Demand-driven Delivery Staff Rostering: Preliminary Results

Andrea Rendl and Christina Burt

Satalia

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# Context of this work: Delivery company

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- Company sells goods that require a manual setup
- Company delivers with their own fleet and staff
- Customers select delivery date and time window

# Cyclic Roster for Drivers

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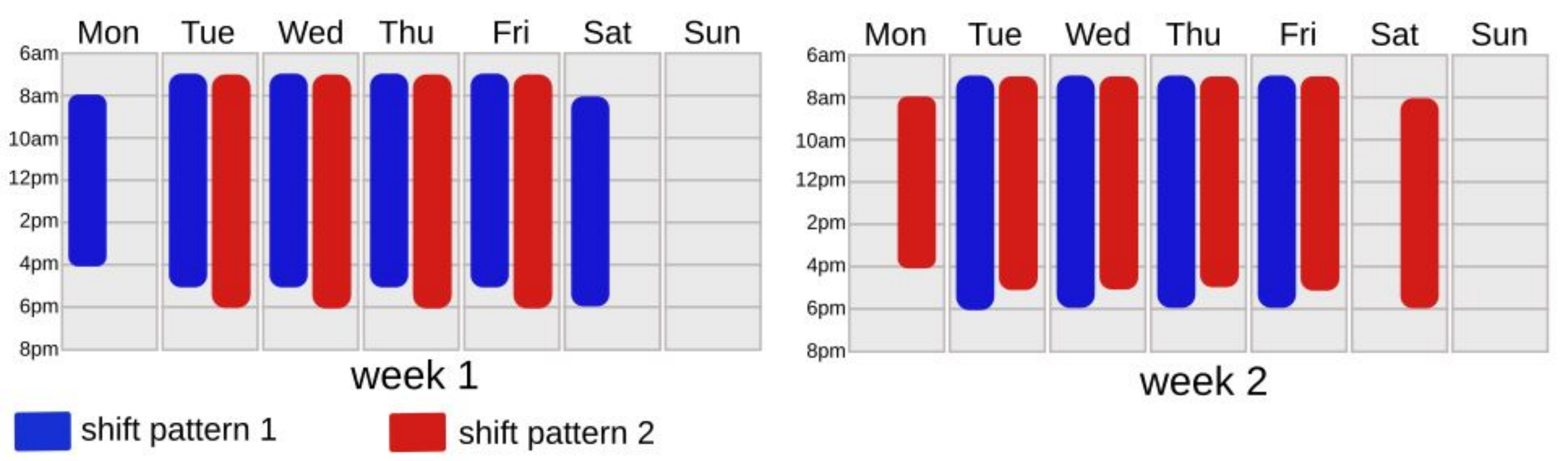

# Van hours and resulting delivery capacity

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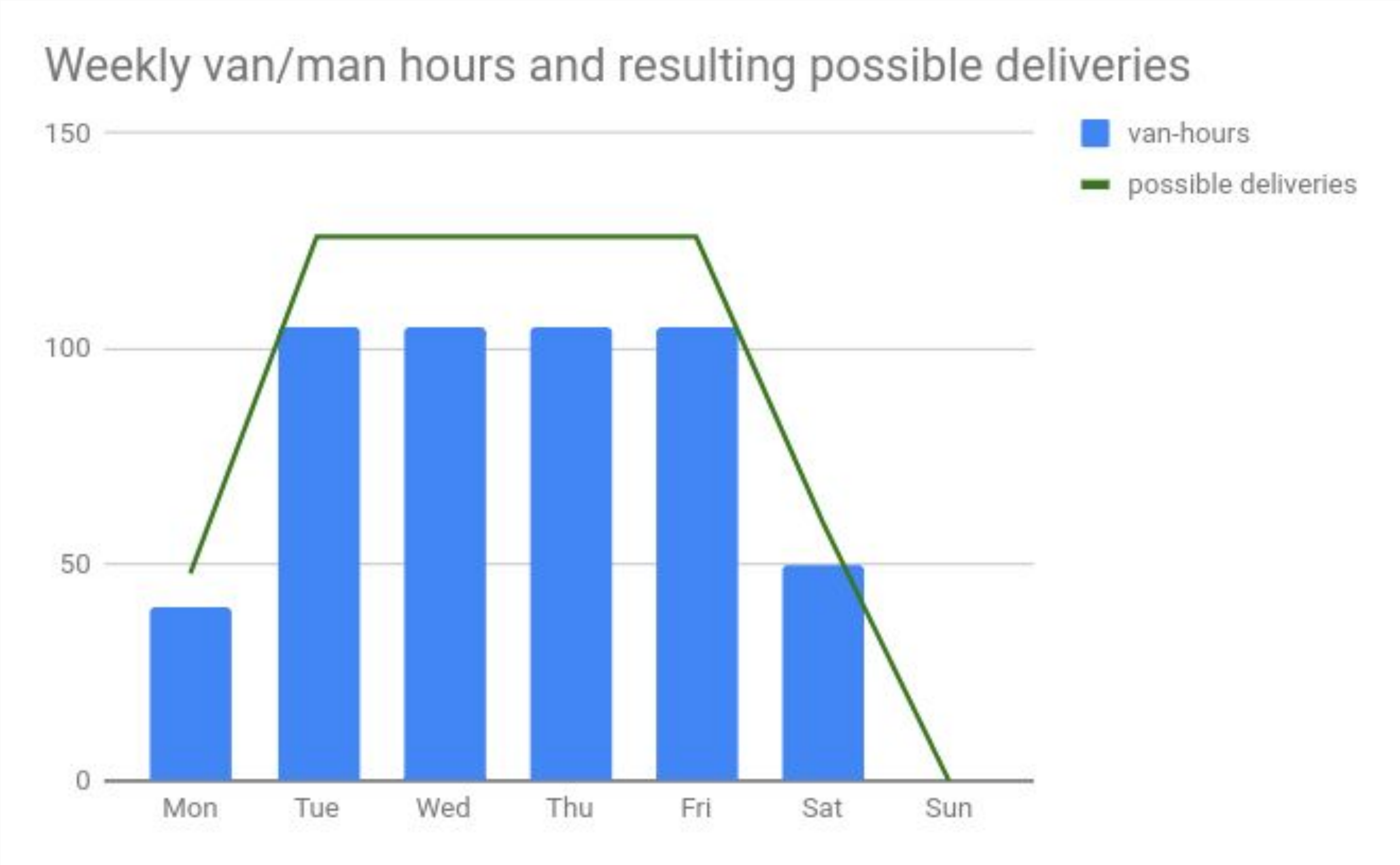
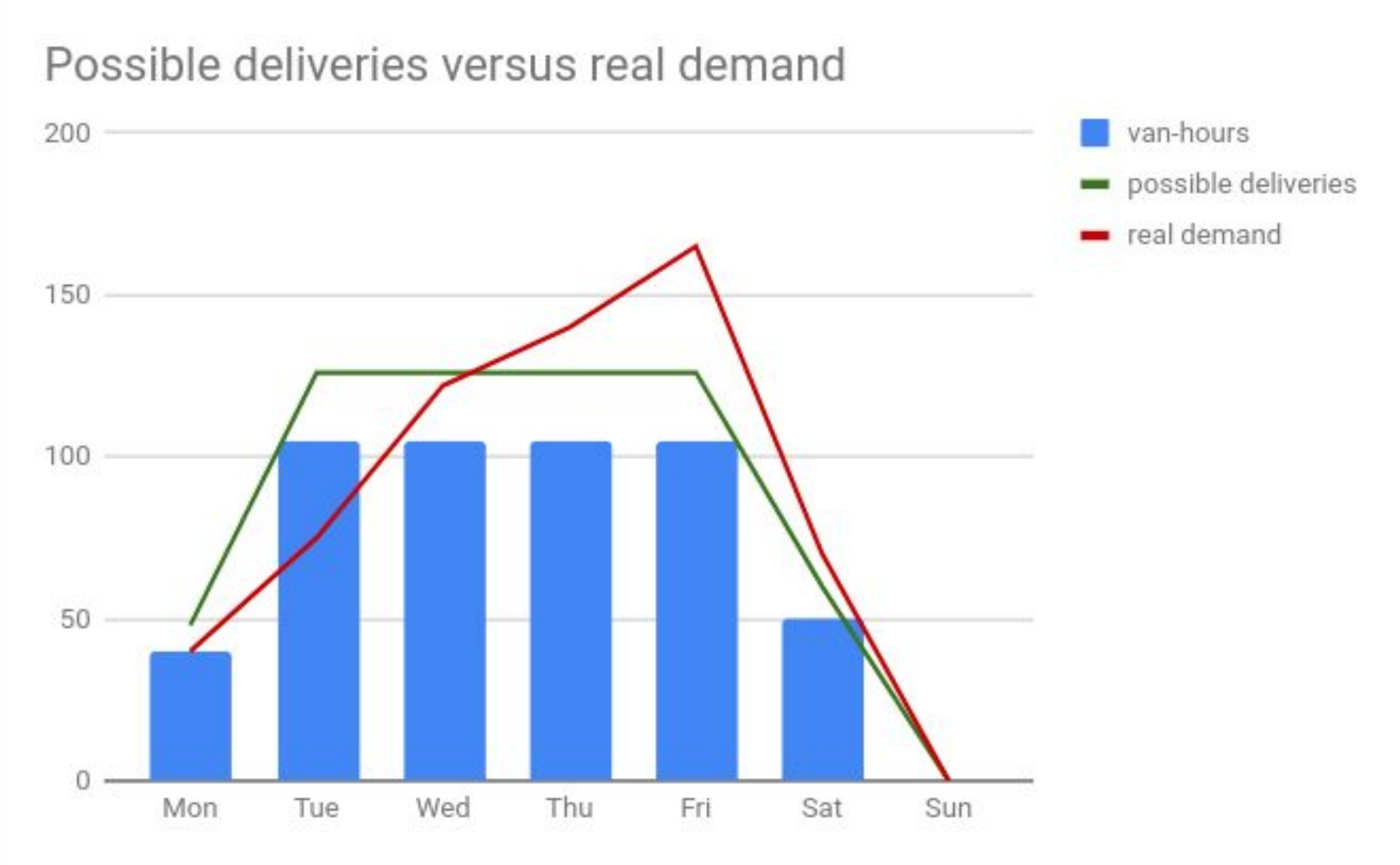
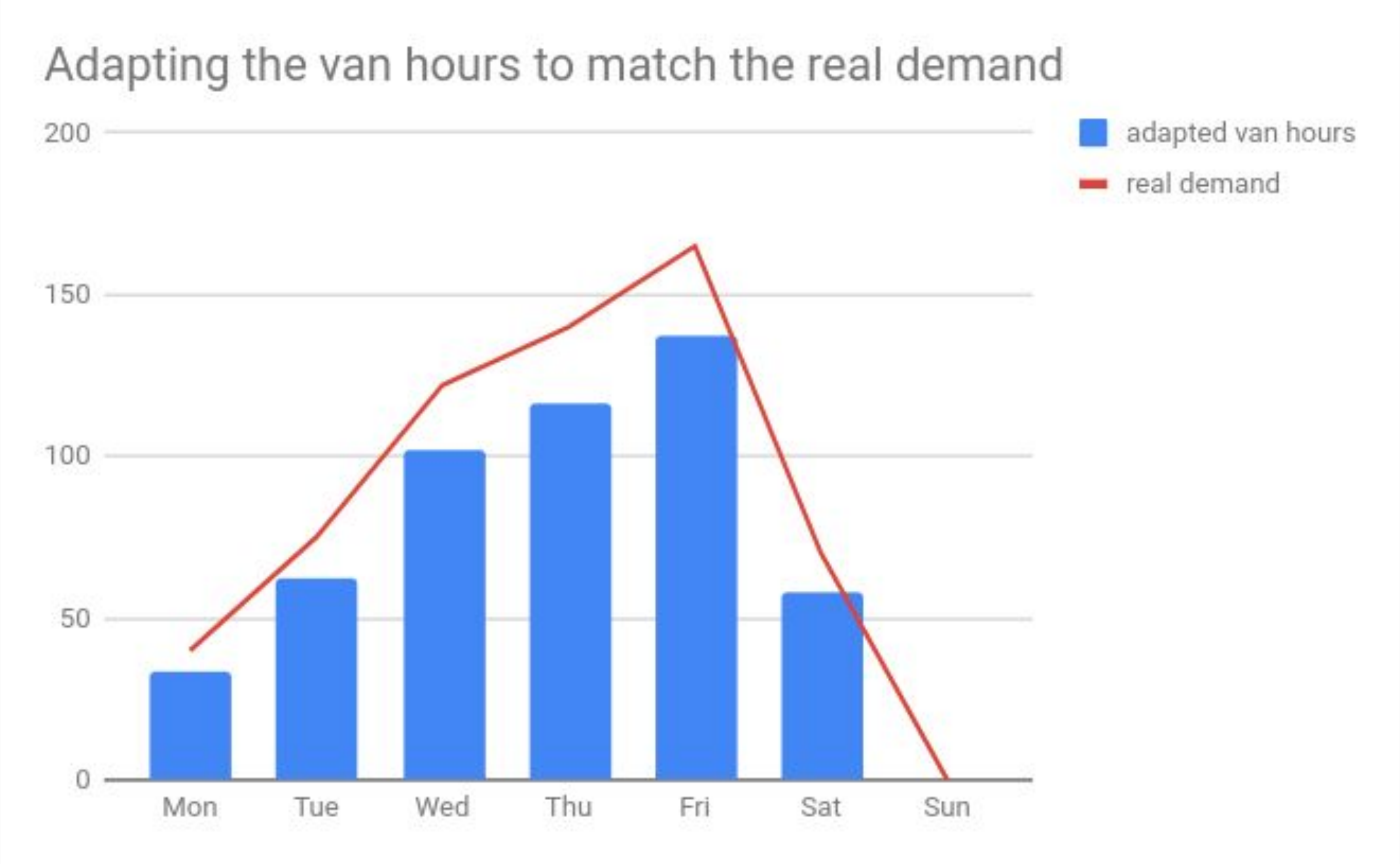


# Problem: capacity from roster does not match demand



# Our goal: find a roster that matches the real demand



# Mathematical Model

# Parameters

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- Shift patterns (weeks) $S$
- Drivers / vans $V$
- Estimated demand per weekday (in orders) $O$

# Constants (1/2)

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- Time factor $\tau$
- Time units $T = \{1..\ 24*\tau\}$
- Stem time $t_{stem}$
- Lunch break duration $t_{lunch}$

# Constants (2/2)

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- Max working hours
  - $t_{daily}$
  - $t_{weekly}$
- Paid working hours $t_{paid}$
- Shift constants
  - min/max shift length
  - Earliest start time
  - Latest end time

- $o$ in $\mathbb{R}+$
  average orders delivered per van per hour
- $vv_v^s$ in {0,1}
  1 if van v is assigned to shift pattern s

# Main Decision variables

- $s^s_d$ in T
  Start time of shift on weekday d, for shift pattern s

- $e^s_d$ in T
  End time of shift on weekday d, for shift pattern s

# Helper Decision variables

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- $l^s_d$ in T
  length of shift on weekday d, for shift pattern s

- $w^s_d$ in {0, 1}
  1 if weekday d in shift pattern s is a working day

- $vh_d$ in $\{0 .. T_{max}\}$
  The number of hours all vans are working on weekday d

# “Objective” decision variables

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- $a_d$ in $\mathbb{R}+$
  The average number of orders delivered on weekday d (over all shift patterns)

- $u_d$ in $\mathbb{R}+$
  Unmet demand (in orders) on weekday d, over all shift patterns

# Shift Constraints

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- $s^s_d \geq earliestStartTime \quad \forall\, s,d$
- $e^s_d \leq latestEndTime \quad \forall\, s,d$
- $e^s_d \geq s^s_d \quad \forall\, s,d$
- $l^s_d = e^s_d - s^s_d \quad \forall\, s,d$
- $l^s_d \leq M * w^s_d \quad \forall\, s,d$ with $M \geq t_{day}$

# Working hour Constraints

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- $\sum_{s,d} \{l^s_d\} - \sum_{s,d} \{w^s_d * t_{lunch}\} = t_{paid}$
  The average number of working hours over all shift patterns must be equal to the number of paid hours

- $\sum_{d} \{l^s_d\} - \sum_{d} \{w^s_d * t_{lunch}\} \leq t_{week} \quad \forall s \text{ in } S$
  For each shift pattern, the maximal number of working hours is not exceeded

# 2-day break Constraints

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- $(w^{s}_{Sat} + w^{s}_{Sun} = 0) + (w^{s}_{Sun} + w^{s+1}_{Mon} = 0)$

  $+ (w^{s}_{Mon} + w^{s}_{Tue} = 0) = 1 \qquad \forall\ s \text{ in } S - 1$

- $(w^{S}_{Sat} + w^{S}_{Sun} = 0) + (w^{S}_{Sun} + w^{1}_{Mon} = 0)$

  $+ (w^{S}_{Mon} + w^{S}_{Tue} = 0) = 1$

There is a two day break between each shift

# Van hour Constraints

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- $vh_d = \sum_{s,v} \{vv^s_v * l^s_d\}$

  $- \quad \sum_{s,d} \{w^s_d\} * \sum_{s,v} \{vv^s_v * t_{lunch}\} \qquad \forall\, d$

  Calculating the van hours $vh_d$ for each weekday d, over all shift patterns

- $vv_v^s$ in {0,1}: 1 if van v is assigned to shift pattern s (constant)
- $w^s_d$ in {0, 1}: 1 if weekday d in shift pattern s is a working day

# Serviced-orders Constraints

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- $a_d = o * (vh_d - 2 * t_{stem} * \{ \sum_{s,d} \{w^s_d\} * \sum_{s,v} \{vv^s_v\} \} ) \quad \forall d$

  Calculating the average number of serviced orders (fleet capacity) $a_d$ for each weekday d: multiplying o with the net worked hours (removing the stem time)

- o : average orders delivered per van per hour
- $vv_v^s$ in {0,1}: 1 if van v is assigned to shift pattern s (constant)
- $w^s_d$ in {0, 1}: 1 if weekday d in shift pattern s is a working day

# Unmet demand Constraints

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- $u_d = | O_d - a_d | \quad \forall d$

  The unmet demand $u_d$ : the absolute value of expected order $O_d$ minus the fleet capacity $a_d$

- $O_d$ in $\mathbb{R}$: expected number of orders on day d
- $a_d$ in $\mathbb{R}$: average fleet capacity in number of orders

# Objective 1: minimize unmet demand

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- Minimize $p$

  Minimize the maximal unmet demand $p$

- $p \geq 0.0$
  $p \leq$ max demand

- $p > u_d \qquad \forall\, d$

# Objective 2: weighted unmet demand

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- Minimize $\sum_d \{ c_d * u_d \}$

  Minimize the unmet demand $u_d$ weighted with $c_d$

# Preliminary Results

# MiniZinc model



- Implemented model in MiniZinc
- Model + data available on github (MIT license):

https://github.com/angee/demand-shift-pattern

(link is also in the paper)

# Problem instances

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- Parameters:
  - Vans/drivers: 12, 24, 60
  - Shift patterns: 2, 4, 6
  - 2 Demand scenarios:
    - Linear-increase of demand over week
    - Peak demand on Thu/Fri

- Reflect real-world problem sizes

# Experimental Setup

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- MiniZinc v2.1.7
- Solvers:
  - Gecode
  - COIN-OR cbc
- Timeout: 300 seconds
- Default search

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| Instance | | | Runtime (sec) | | Objective | |
|---|---|---|---|---|---|---|
| vans | shift p. | demand | Gecode | cbc | Gecode | cbc |
| v12 | s6 | linear | 300.000 | 300.000 | — | 3.81728 |
| v24 | s2 | linear | 0.027 | 0.353 | *27.0 | *27.03456 |
| v24 | s4 | linear | 300.000 | 10.006 | — | *27.03456 |
| v60 | s6 | linear | 300.000 | 241.377 | — | *17.0864 |
| v60 | s2 | linear | 0.026 | 0.328 | *68.0 | *68.0864 |
| v60 | s4 | linear | 300.000 | 11.444 | — | *68.0864 |
| v24 | s6 | linear | 300.000 | 300.000 | — | 18.23456 |
| v12 | s2 | linear | 300.000 | 0.317 | 38.2 | *13.01728 |
| v12 | s4 | linear | 300.000 | 17.668 | — | *13.01728 |
| v60 | s4 | peak-thu-fri | 300.000 | 2.404 | — | *42.0864 |
| v12 | s2 | peak-thu-fri | 0.013 | 0.292 | *17.0 | *17.01728 |
| v60 | s2 | peak-thu-fri | 0.014 | 0.327 | *84.0 | *84.0864 |
| v12 | s4 | peak-thu-fri | 300.000 | 2.161 | — | 8.01728 |
| v24 | s2 | peak-thu-fri | 0.047 | 0.284 | *33.0 | *33.03456 |
| v60 | s6 | peak-thu-fri | 300.000 | 0.728 | — | *12.0864 |
| v12 | s6 | peak-thu-fri | 300.000 | 1.16 | — | *2.41728 |
| v24 | s4 | peak-thu-fri | 300.000 | 5.765 | — | *17.03456 |
| v24 | s6 | peak-thu-fri | 300.000 | 0.962 | — | *5.43456 |

# Observations

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- MIP solver outperforms CP solver
  - We do not use full power of CP
    - search strategy
    - global constraints

- Several optimal solutions cannot match demand
  - Working hour settings very conservative

# Future Work

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- Alternative CP-style formulation
    - Global constraints
    - Custom search strategies

- Include optional constraints
    - E.g. holidays every other Saturday

- Evaluate constant settings: with what settings can we find a solution to fully match the demand?