香港中文大學
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Automatic Generation of Dominance Breaking Nogoods for Constraint Optimisation

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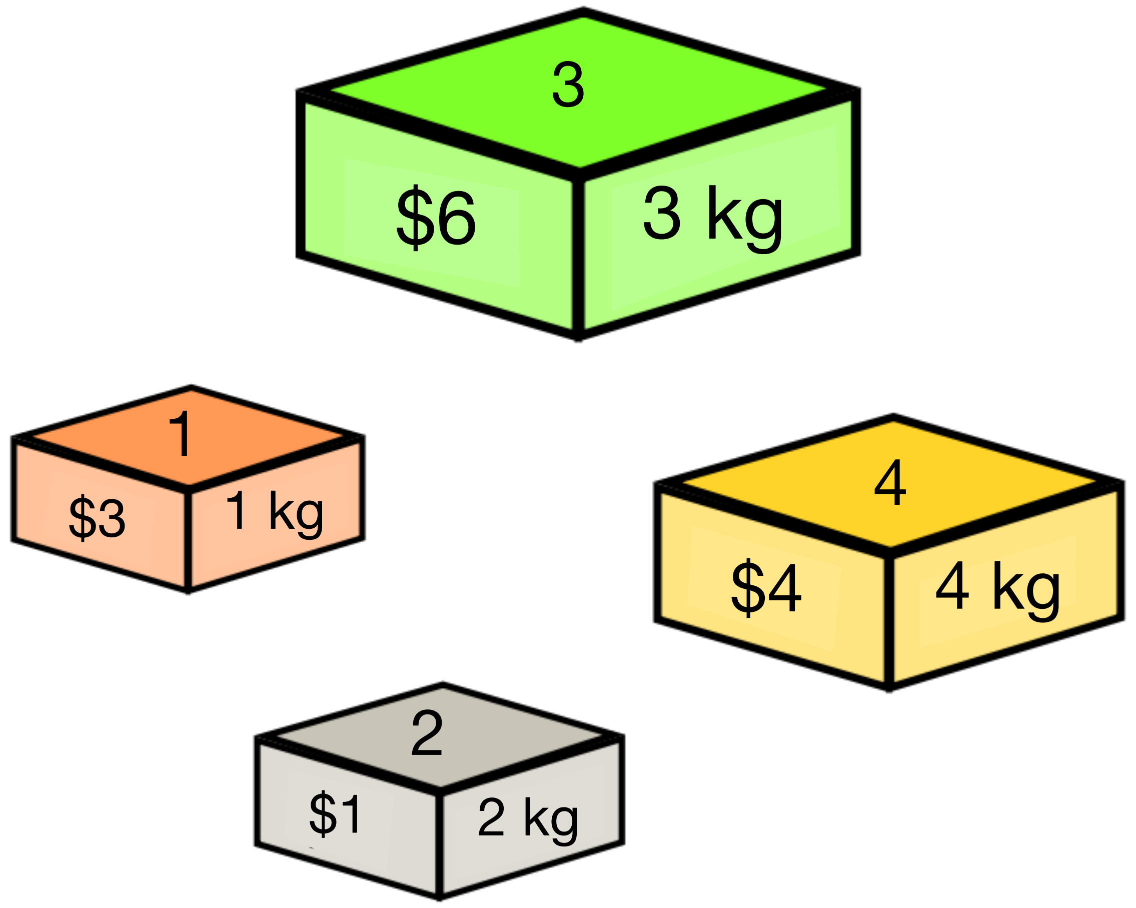
Constraint Optimisation Problems (COPs)

- Example: 0-1 Knapsack
  - Variables: x_1, x_2, x_3, x_4
  - Domains: $x_i \in \{0,1\}$ for $i = 1,\ldots,4$
  - Constraint: $x_1 + 2x_2 + 3x_3 + 4x_4 \leq 5$
  - Objective: maximize $3x_1 + x_2 + 6x_3 + 4x_4$

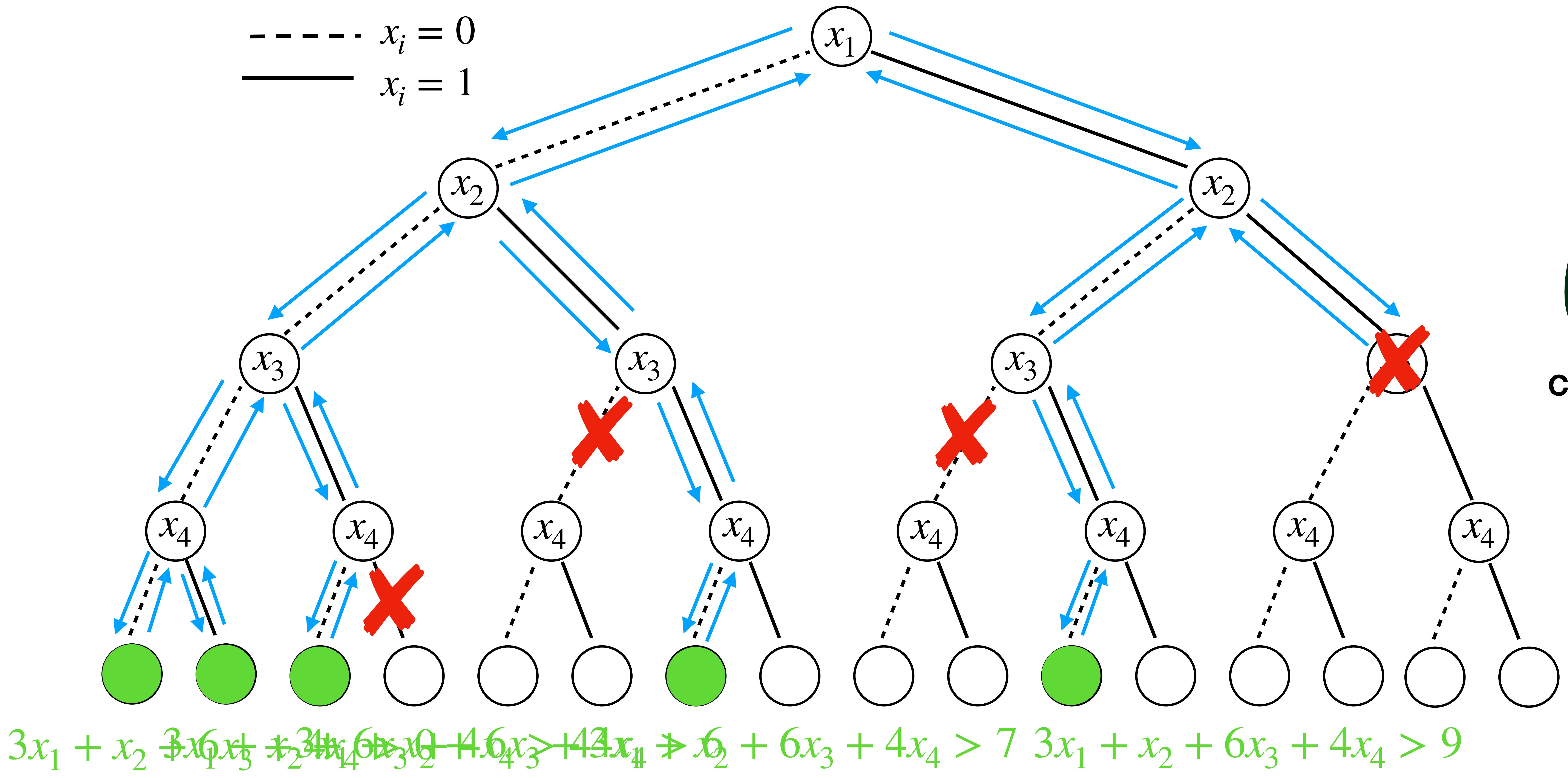
Capacity: 5 kg



- Goal: find an assignment of values to variables such that
  - all constraints are satisfied, and
  - the objective is optimized

Dominance Breaking

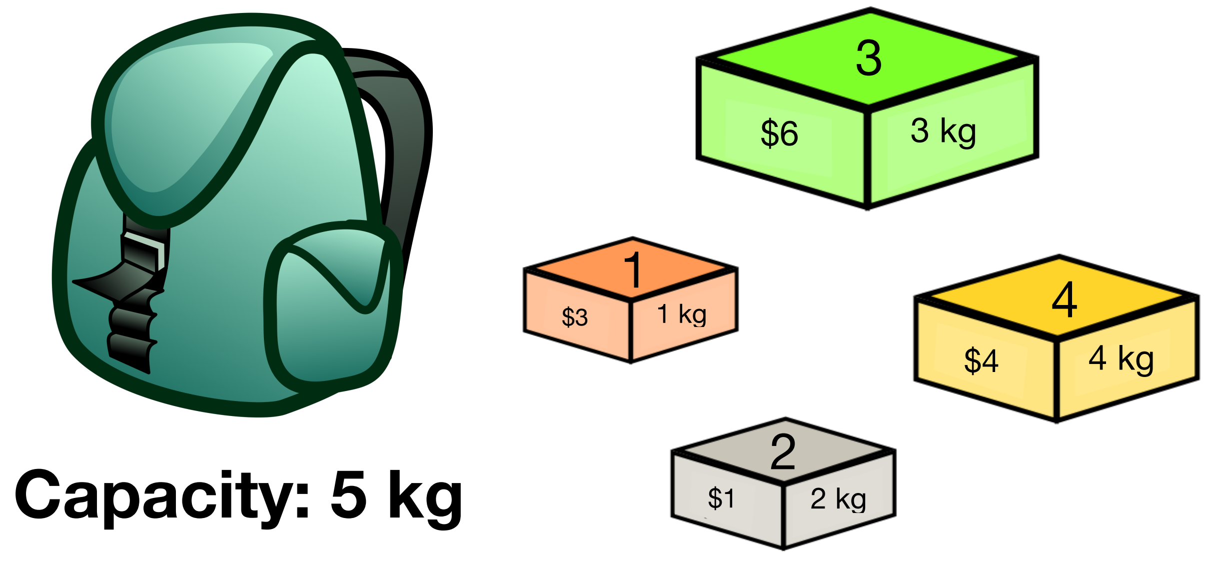
- Branch and bound:



maximize $3x_1 + x_2 + 6x_3 + 4x_4$

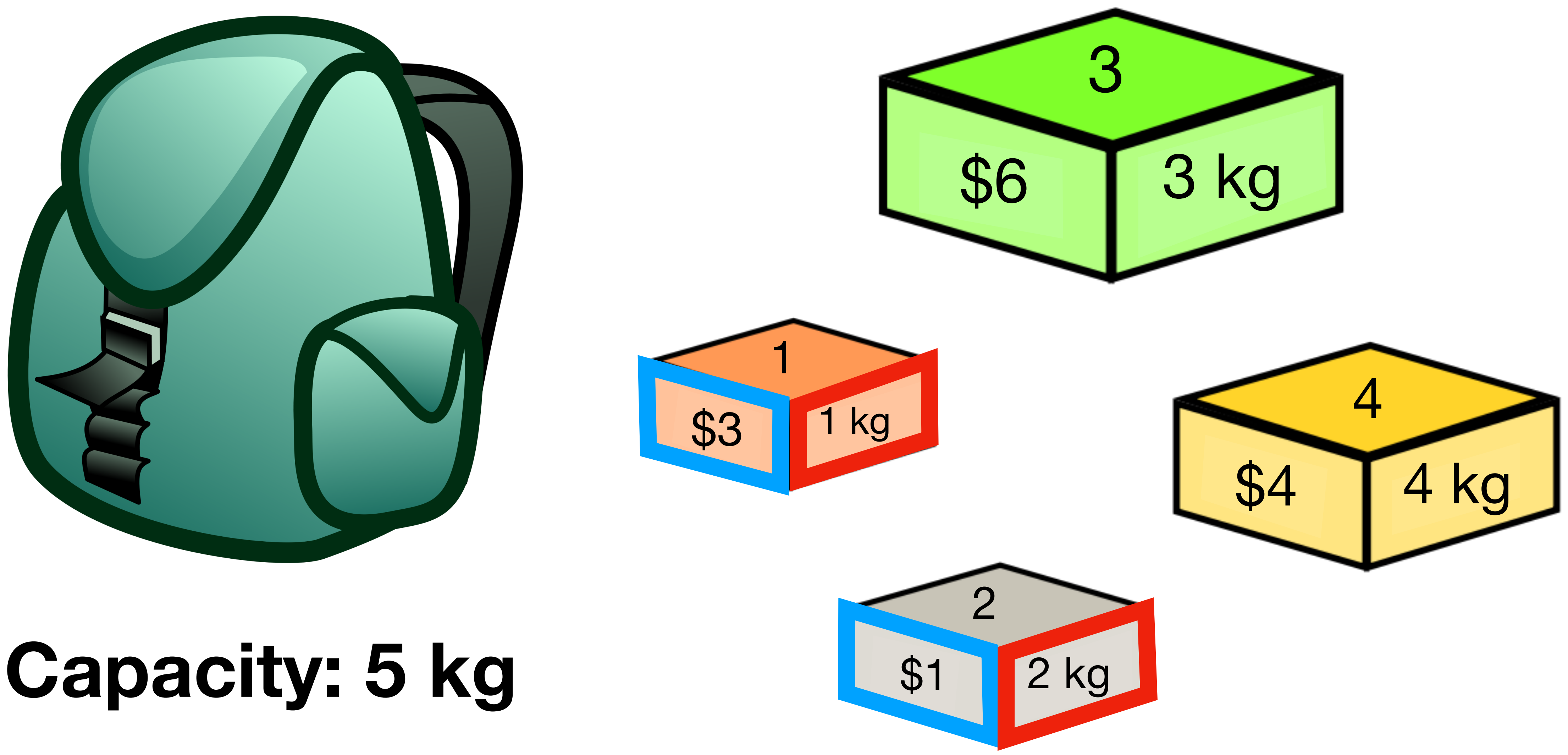
s.t. $x_1 + 2x_2 + 3x_3 + 4x_4 \leq 5$

$x_i \in \{0,1\}$ for $i = 1,\ldots,4$



Dominance Breaking

- Dominance breaking is a technique to prune suboptimal assignments.



Capacity: 5 kg

$$\text{maximize } 3x_1 + x_2 + 6x_3 + 4x_4$$

$$\text{s.t. } x_1 + 2x_2 + 3x_3 + 4x_4 \leq 5$$

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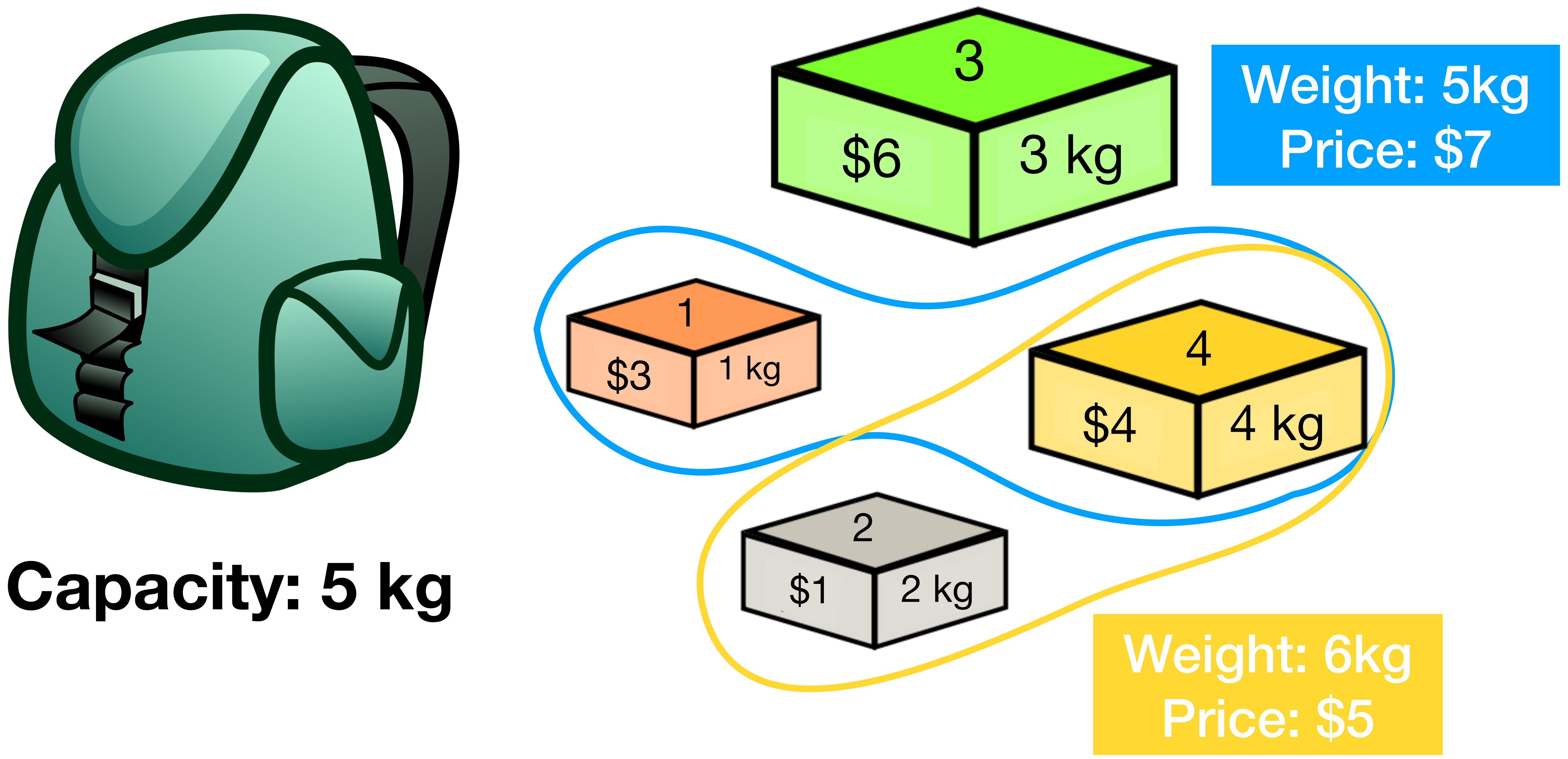


maximize $3x_1 + x_2 + 6x_3 + 4x_4$

s.t. $x_1 + 2x_2 + 3x_3 + 4x_4 \leq 5$

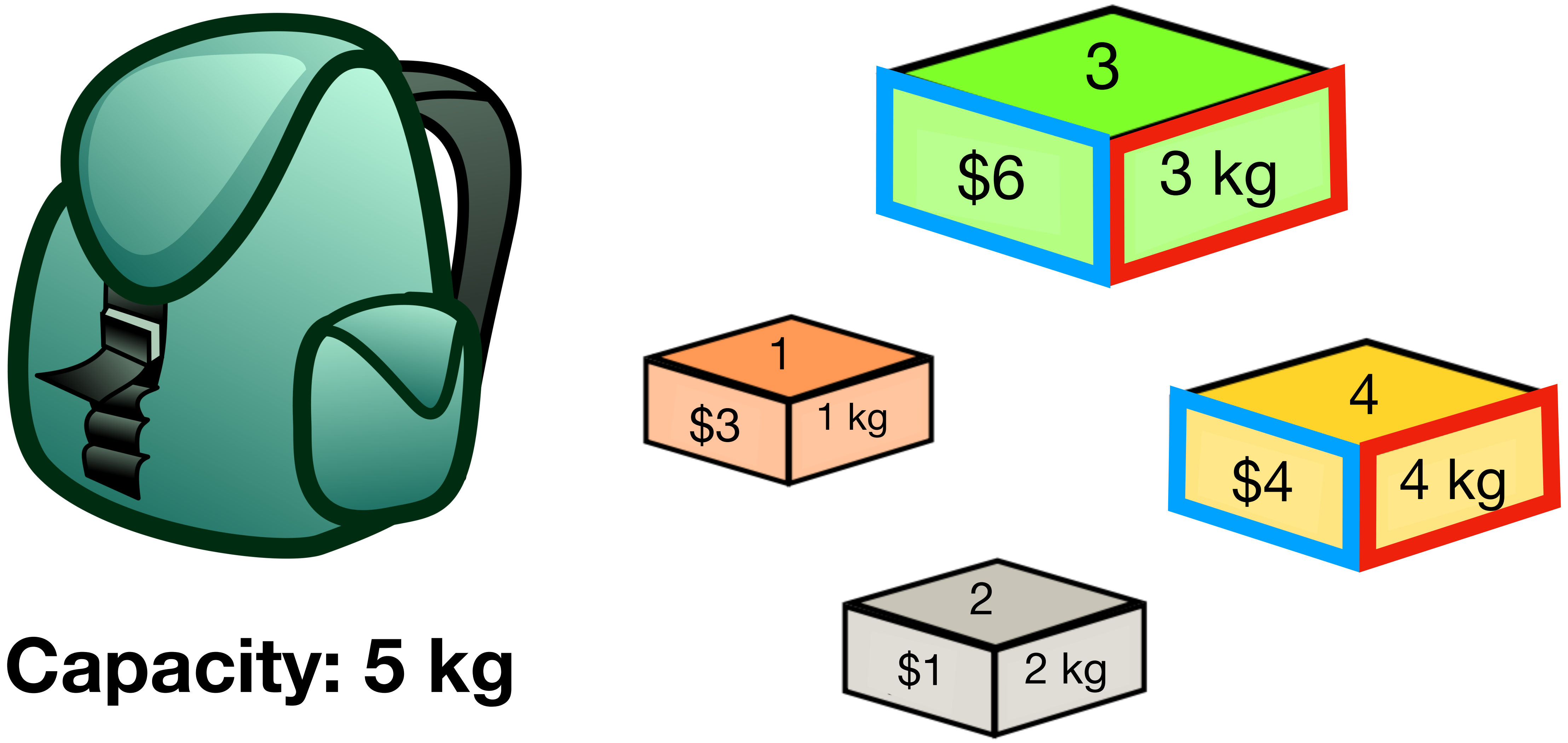
$x_i \in \{0,1\}$ for $i = 1,\ldots,4$

Observation:

Any assignment selecting item 2 but not item 1 must be suboptimal.

Dominance Breaking

- Dominance breaking is a technique to prune suboptimal assignments.



Capacity: 5 kg

maximize $3x_1 + x_2 + 6x_3 + 4x_4$

s.t. $x_1 + 2x_2 + 3x_3 + 4x_4 \leq 5$

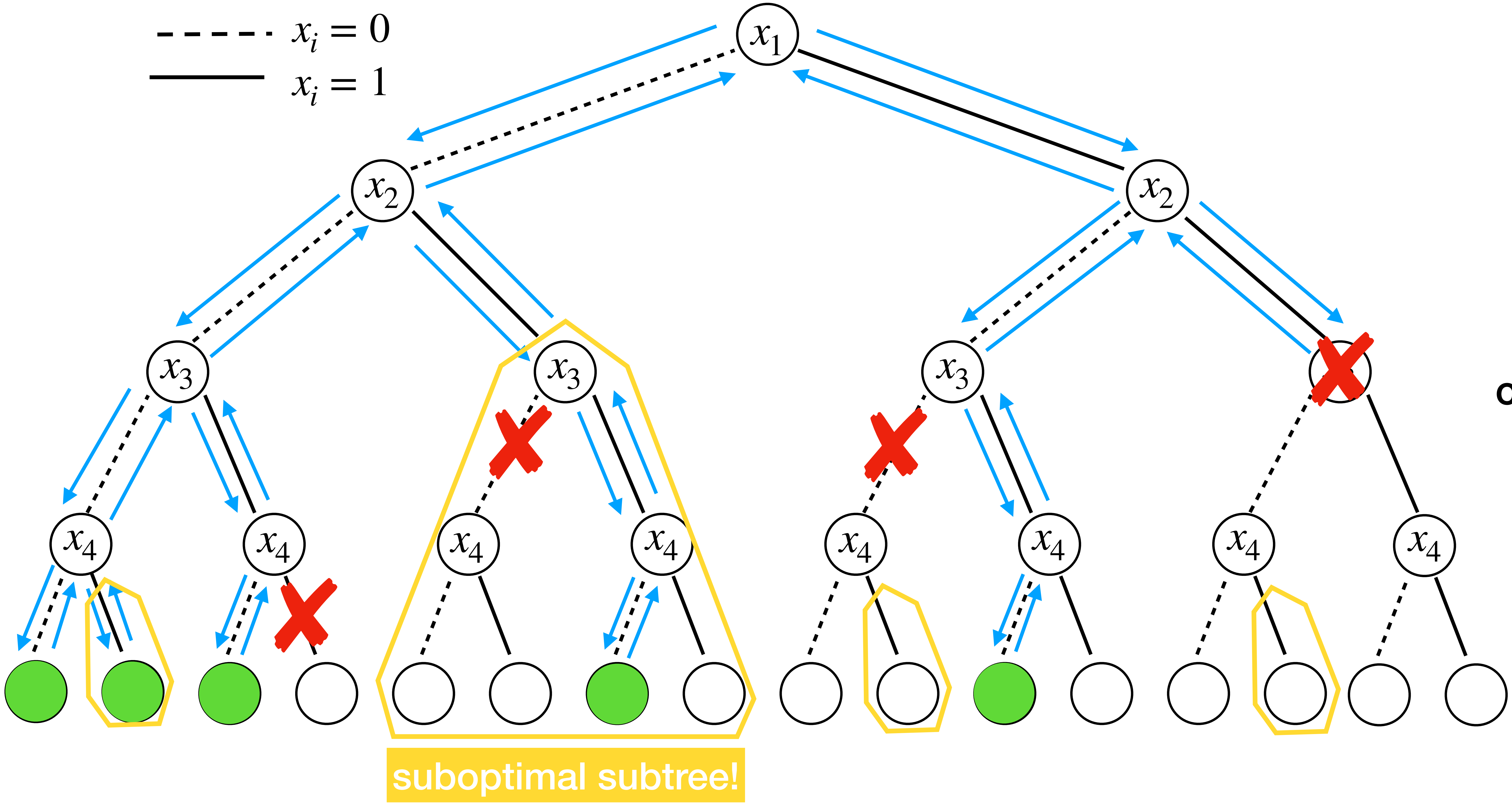
$x_i \in \{0,1\}$ for $i = 1,\ldots,4$

$\boxed{x_2 \leq x_1,\ x_4 \leq x_3}$

Dominance breaking constraints

Dominance Breaking

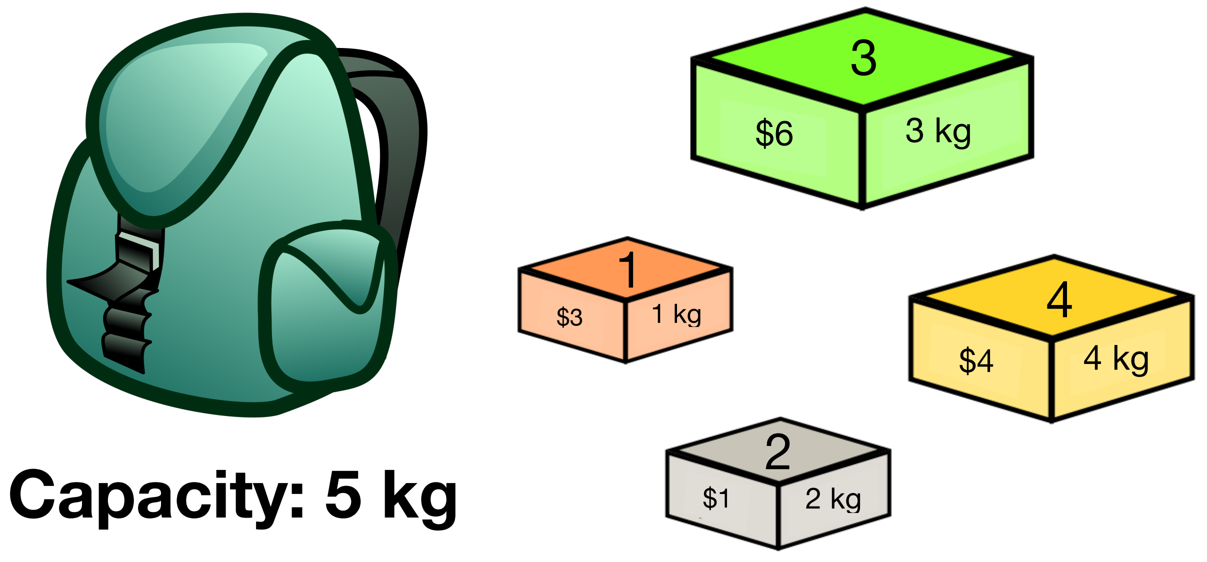
- Branch and bound:



maximize $3x_1 + x_2 + 6x_3 + 4x_4$

s.t. $x_1 + 2x_2 + 3x_3 + 4x_4 \leq 5$

$x_i \in \{0,1\}$ for $i = 1,\ldots,4$



$$\boxed{x_2 \leq x_1,\ x_4 \leq x_3}$$

Dominance breaking constraints

Dominance Breaking

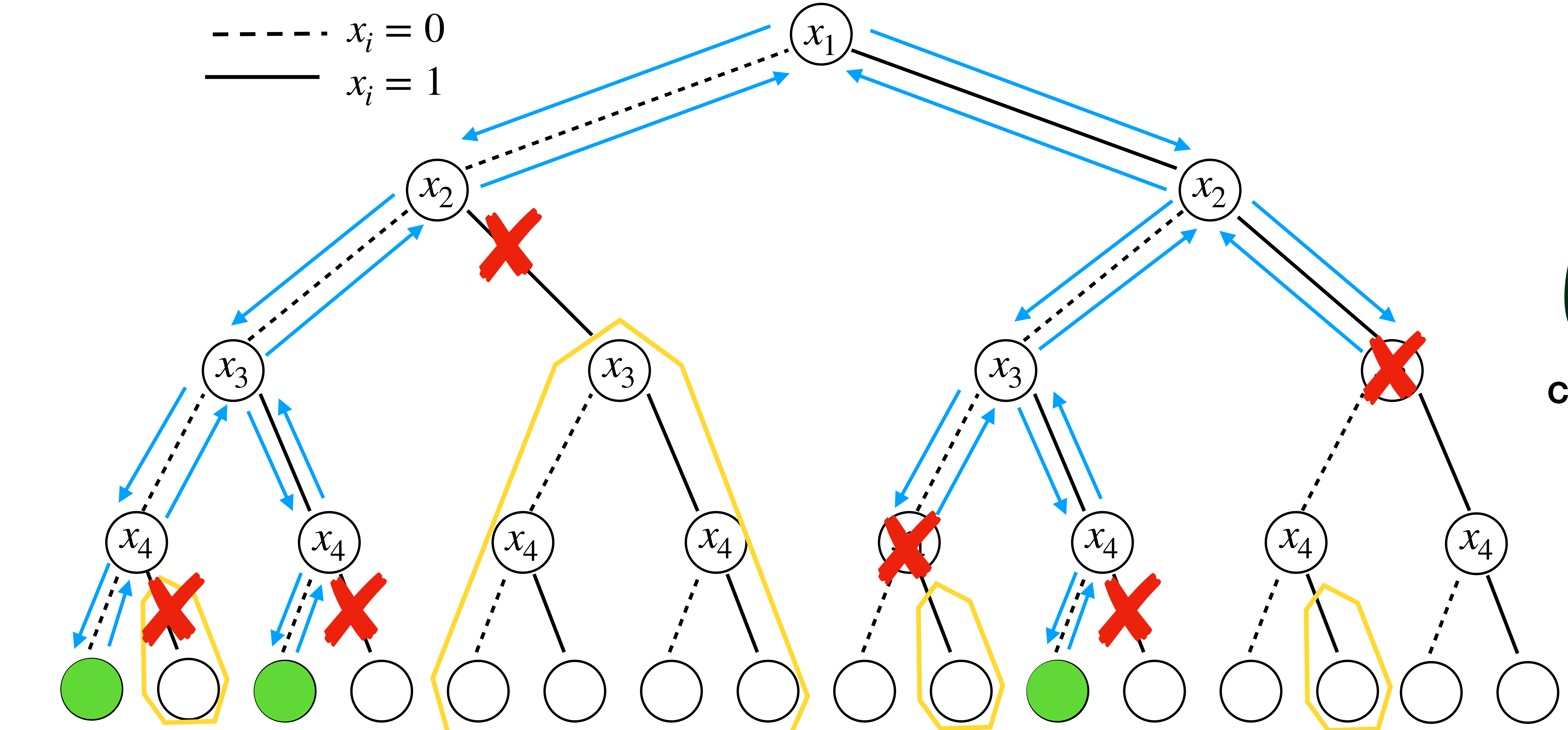
- Branch and bound:

$$\text{maximize } 3x_1 + x_2 + 6x_3 + 4x_4$$

$$\text{s.t. } x_1 + 2x_2 + 3x_3 + 4x_4 \le 5$$

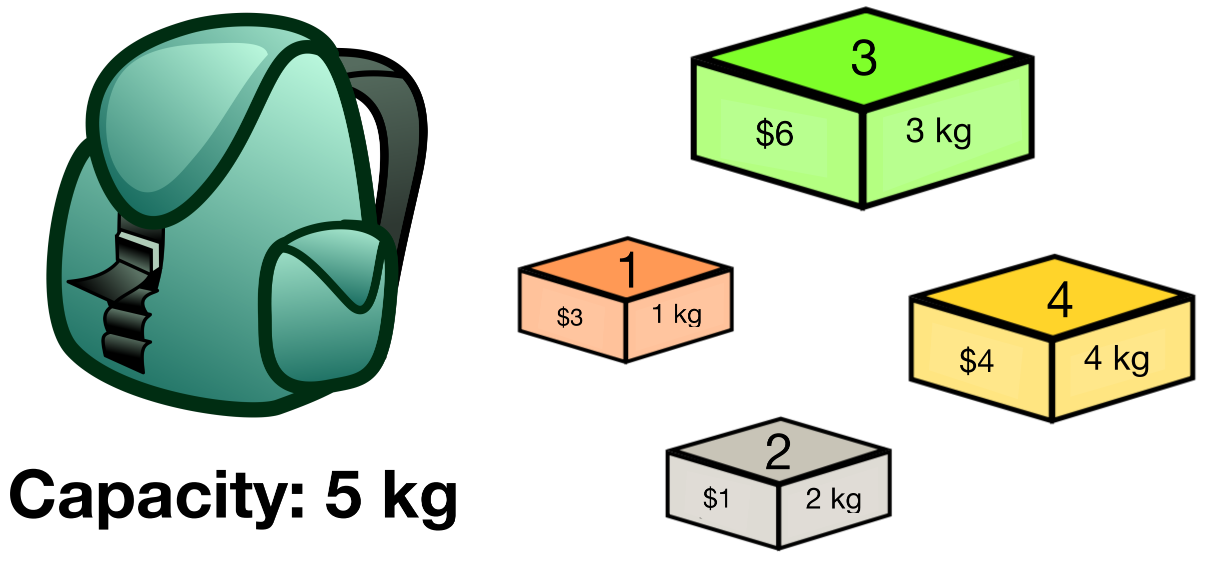
$$x_2 \le x_1,\ x_4 \le x_3$$

$$x_i \in \{0,1\} \text{ for } i = 1,\ldots,4$$



$3x_1 + x_2 + 6x_3 + 4x_4 > 0$ $3x_1 + x_2 + 6x_3 + 4x_4 > 6$

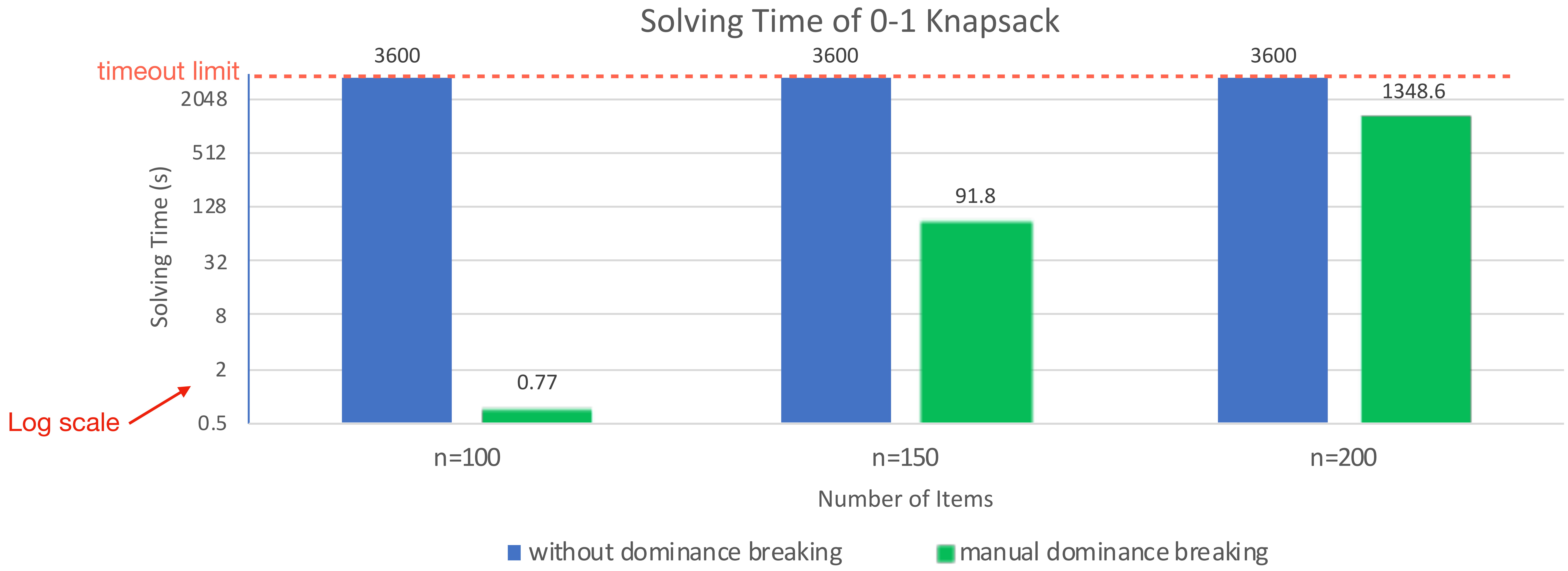
$3x_1 + x_2 + 6x_3 + 4x_4 > 9$



Capacity: 5 kg

Dominance Breaking

- Preliminarily results using the Chuffed solver

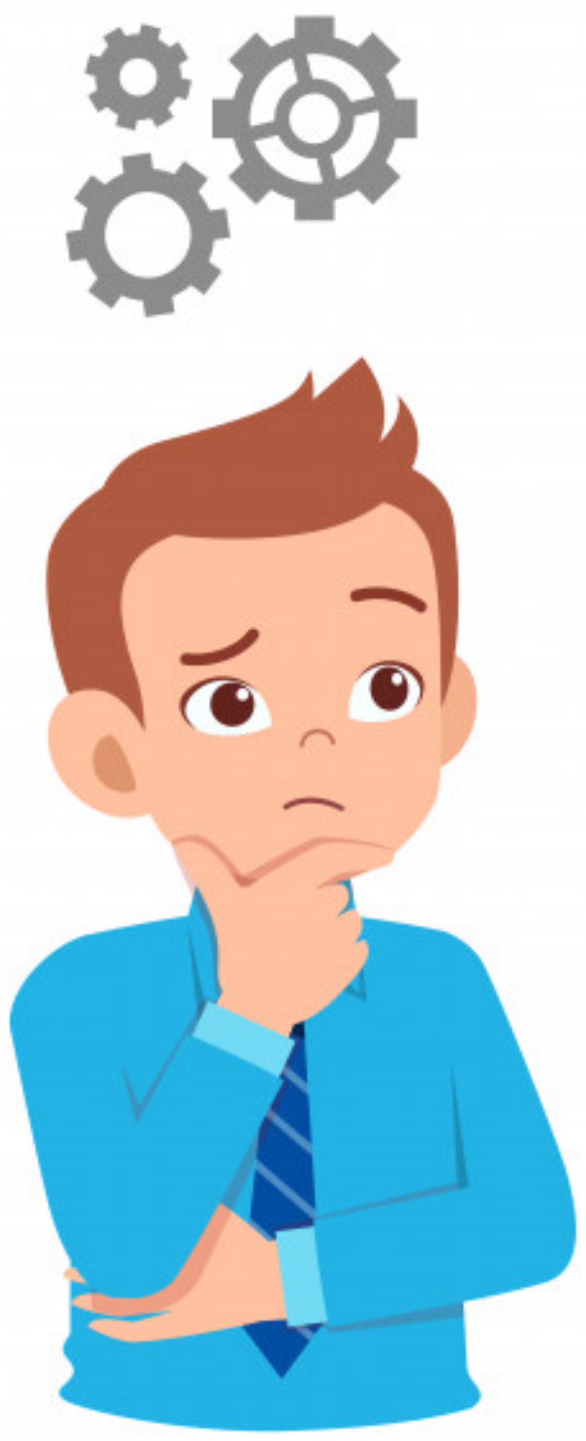


Data source: https://people.eng.unimelb.edu.au/pstuckey/dominance/

Dominance Breaking

- Dominance breaking has been applied successfully in solving many COPs
  - Knapsack problem (Poirriez et al. 2009)
  - Packing problems: rectangle packing (Korf 2004), multicontainer packing (Fukunaga and Korf 2007)
  - Sequencing problems: talent scheduling (Qin et al. 2016, Garcia de la Banda et al. 2011), travelling salesman with time window (Baldacci et al. 2012), minimisation of open stack (Chu et al. 2009)
  - Scheduling problems: balanced academic curriculum problem (Monette et al. 2007), engineer service delivery (Ilankaikone et al. 2021)

Motivation



Problem Model → Model with dominance breaking

Different problems,
Different dominance breaking constraints

Our Approach

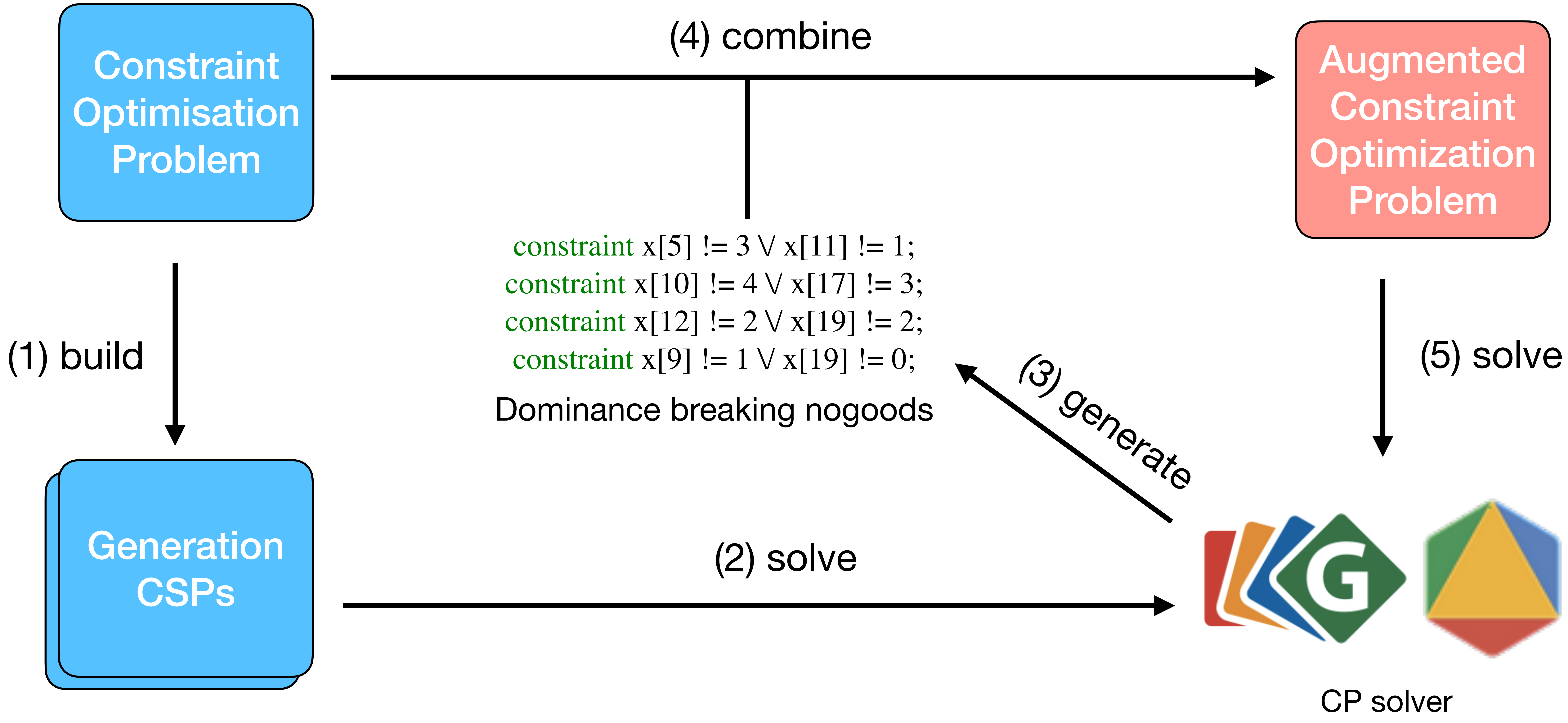
- Focus on nogood constraints
- Full automation
- Solver independence
- More dominance breaking constraints than human
- More efficient than manual methods

Automatic Dominance Breaking

- A formal framework for automatic dominance breaking for a class of constraint optimisation problems
    - Automatic Dominance Breaking for a Class of Constraint Optimization Problems. Jimmy H.M. Lee and **Allen Z. Zhong**, IJCAI-PRICAI 2020
- More efficient generation of dominance breaking nogoods
    - Towards More Practical and Efficient Automatic Dominance Breaking. Jimmy H.M. Lee and **Allen Z. Zhong**, AAAI 2021
- Handling more complex and flexible problems
    - Exploiting Functional Constraints in Generating Dominance Breaking Nogoods for Constraint Optimization. Jimmy H.M. Lee and **Allen Z. Zhong**, CP 2022

Automation Pipeline

(Lee and Zhong 2020)

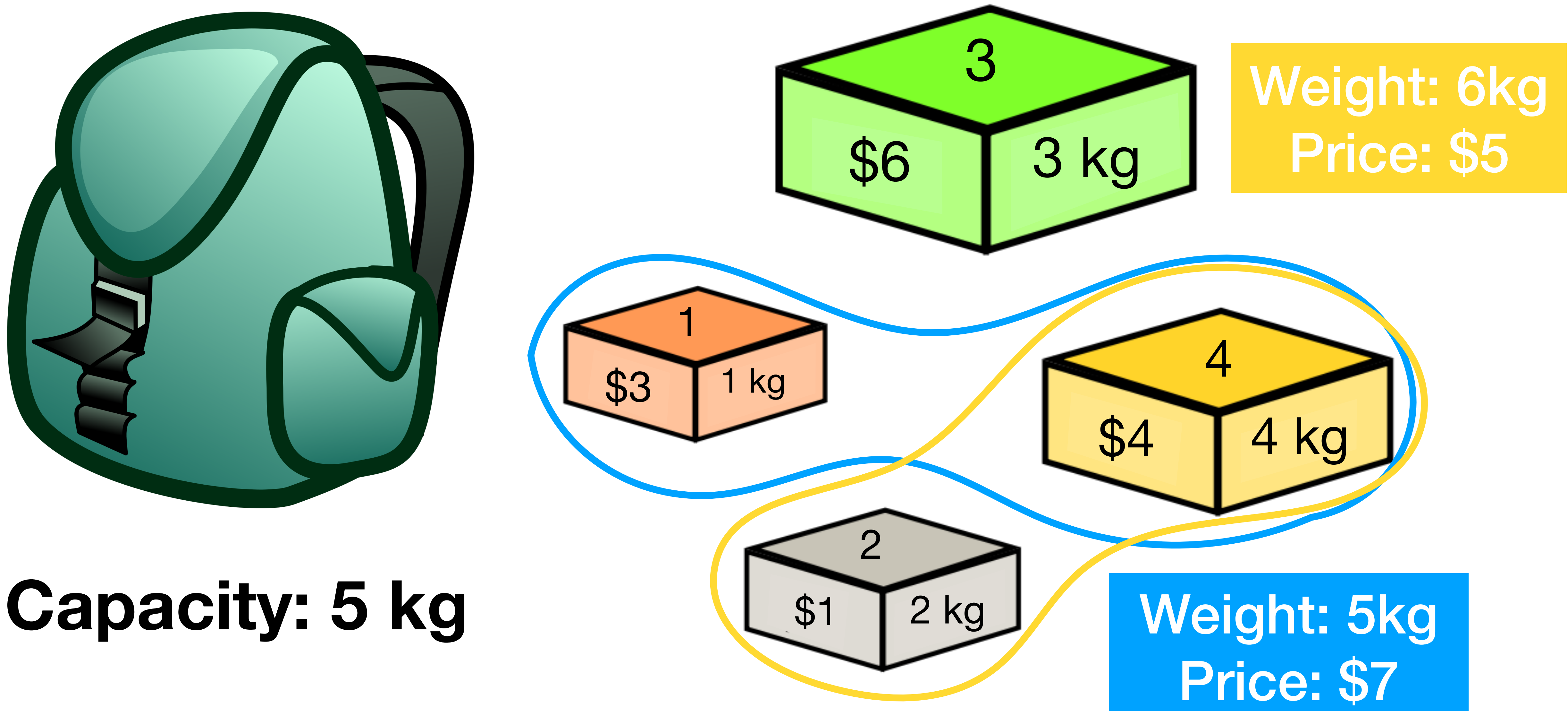


Dominance Relations Over Full Assignments

(Chu and Stuckey 2012)

$\bar{\theta}$ dominates $\bar{\theta}'$ ($\bar{\theta} \prec \bar{\theta}'$):

- $\bar{\theta}$ solution, $\bar{\theta}'$ non-solution



$$\boxed{\bar{\theta} = \{x_1 = 1,\ x_2 = 0,\ x_3 = 0,\ x_4 = 1\}} \prec \boxed{\bar{\theta}' = \{x_1 = 0,\ x_2 = 0,\ x_3 = 1,\ x_4 = 1\}}$$

solution

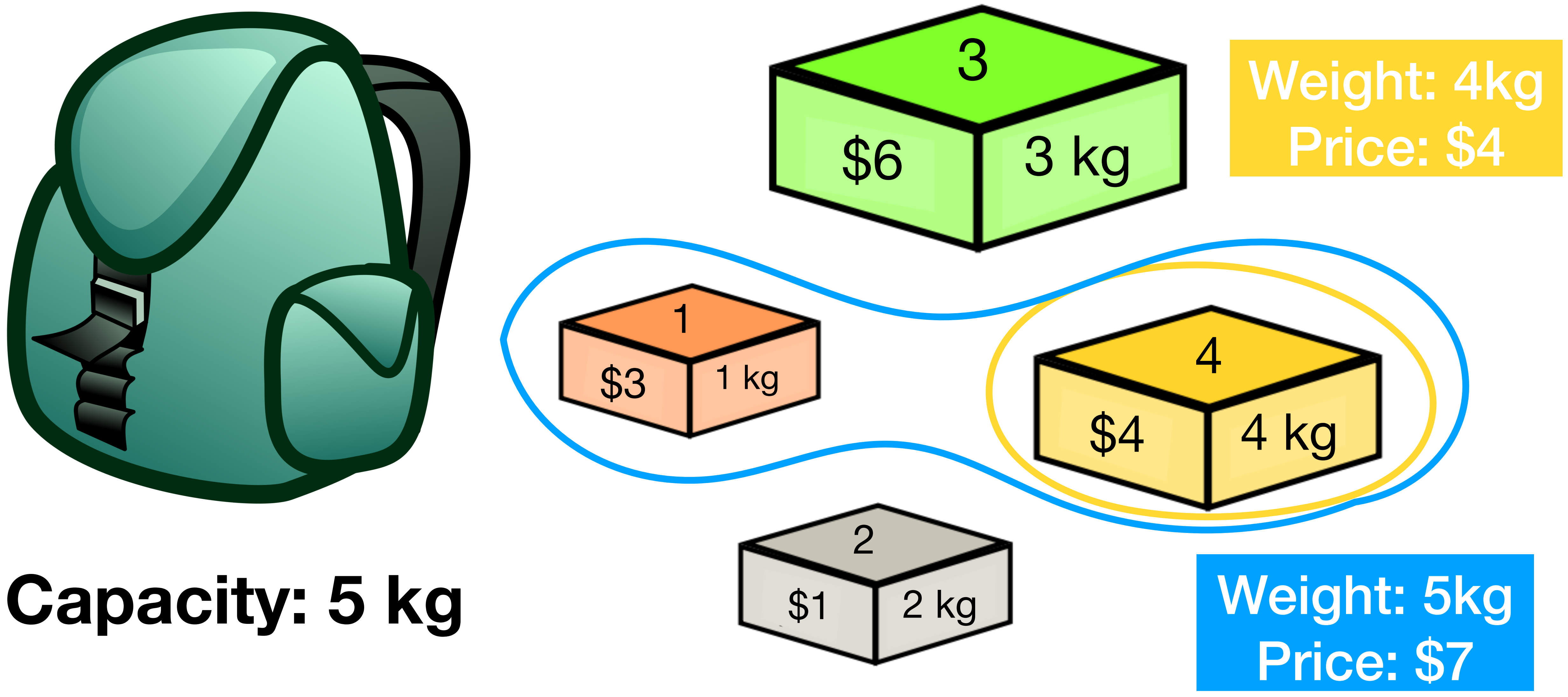
non-solution

Dominance Relations Over Full Assignments

(Chu and Stuckey 2012)

$\bar{\theta}$ dominates $\bar{\theta}'$ ($\bar{\theta} \prec \bar{\theta}'$):

- $\bar{\theta}$ solution, $\bar{\theta}'$ non-solution
- both solutions/non-solutions, $f(\bar{\theta})$ is better than $f(\bar{\theta}')$



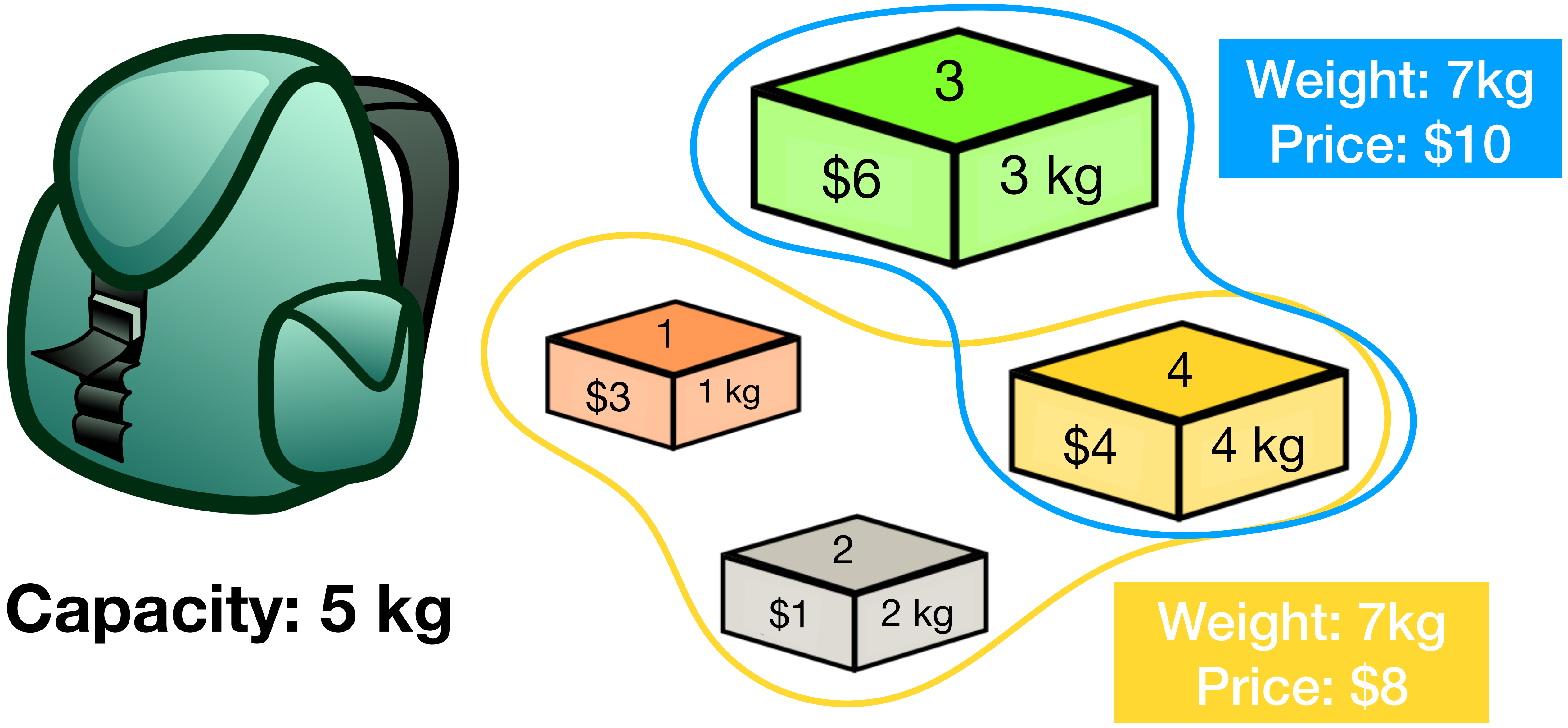
$$\boxed{\bar{\theta} = \{x_1 = 1,\, x_2 = 0,\, x_3 = 0,\, x_4 = 1\}} \prec \boxed{\bar{\theta}' = \{x_1 = 0,\, x_2 = 0,\, x_3 = 0,\, x_4 = 1\}}$$

solution $f(\bar{\theta}) = 7$ $f(\bar{\theta}') = 4$ solution

Dominance Relations Over Full Assignments

(Chu and Stuckey 2012)

if $\bar{\theta}'$ is dominated (by some $\bar{\theta}$), we can safely remove $\bar{\theta}'$



$\bar{\theta} = \{x_1 = 0,\, x_2 = 0,\, x_3 = 1,\, x_4 = 1\}$ $\prec$ $\bar{\theta}' = \{x_1 = 1,\, x_2 = 1,\, x_3 = 0,\, x_4 = 1\}$

non-solution $f(\bar{\theta}) = 10$ $f(\bar{\theta}') = 8$ non-solution

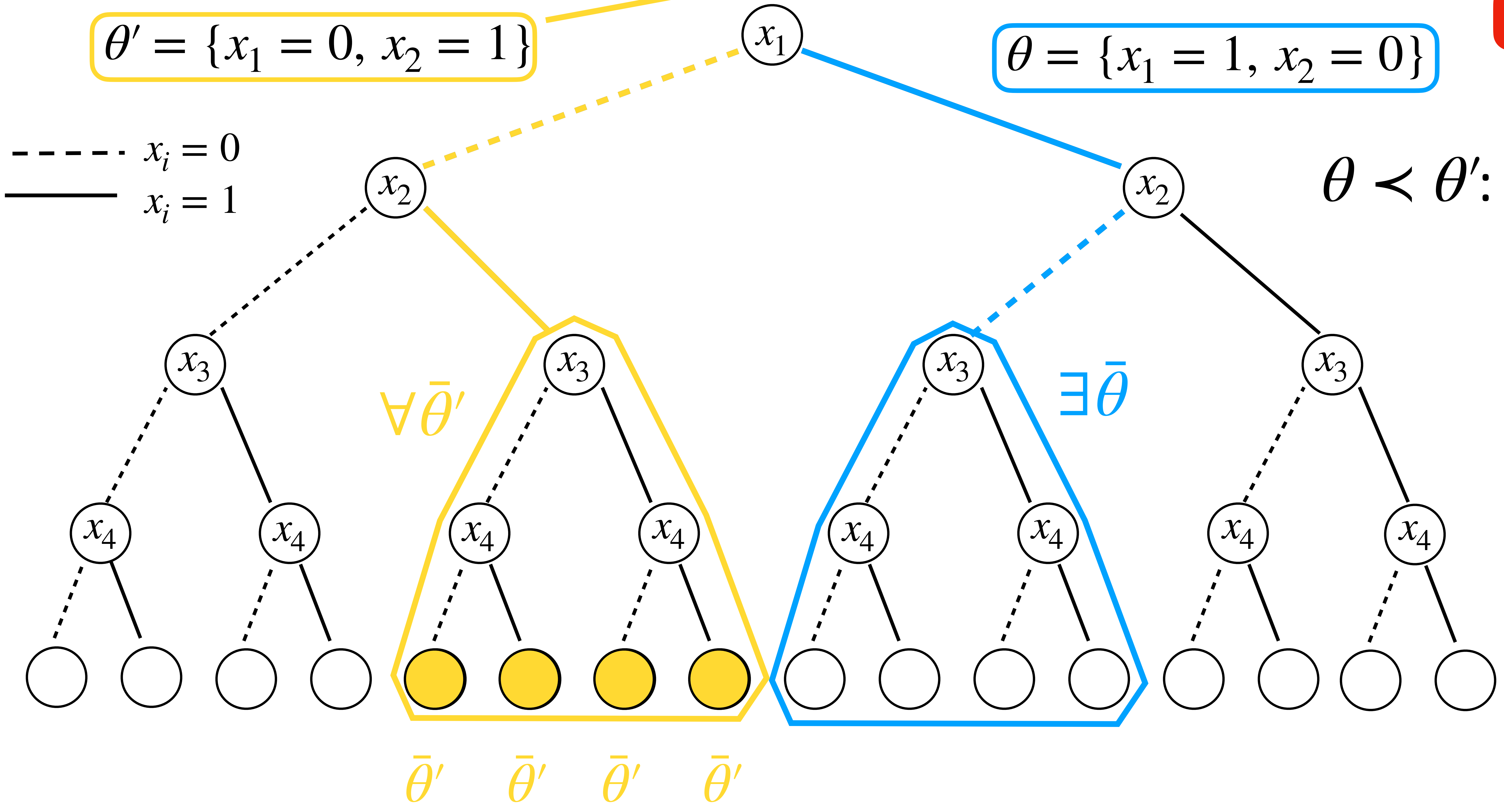
Dominance Relations Over Partial Assignments

(Lee and Zhong 2020)

$\theta' = \{x_1 = 0, x_2 = 1\}$ $\xrightarrow{\text{equivalent}}$ $(x_1 = 0 \land x_2 = 1)$ $\xrightarrow{\text{negation}}$ $\boxed{\neg\theta' \equiv x_1 \neq 0 \lor x_2 \neq 1}$

Dominance Breaking Nogood

$\theta = \{x_1 = 1, x_2 = 0\}$



$\theta \prec \theta'$: $\forall\bar{\theta}'$ extending θ'

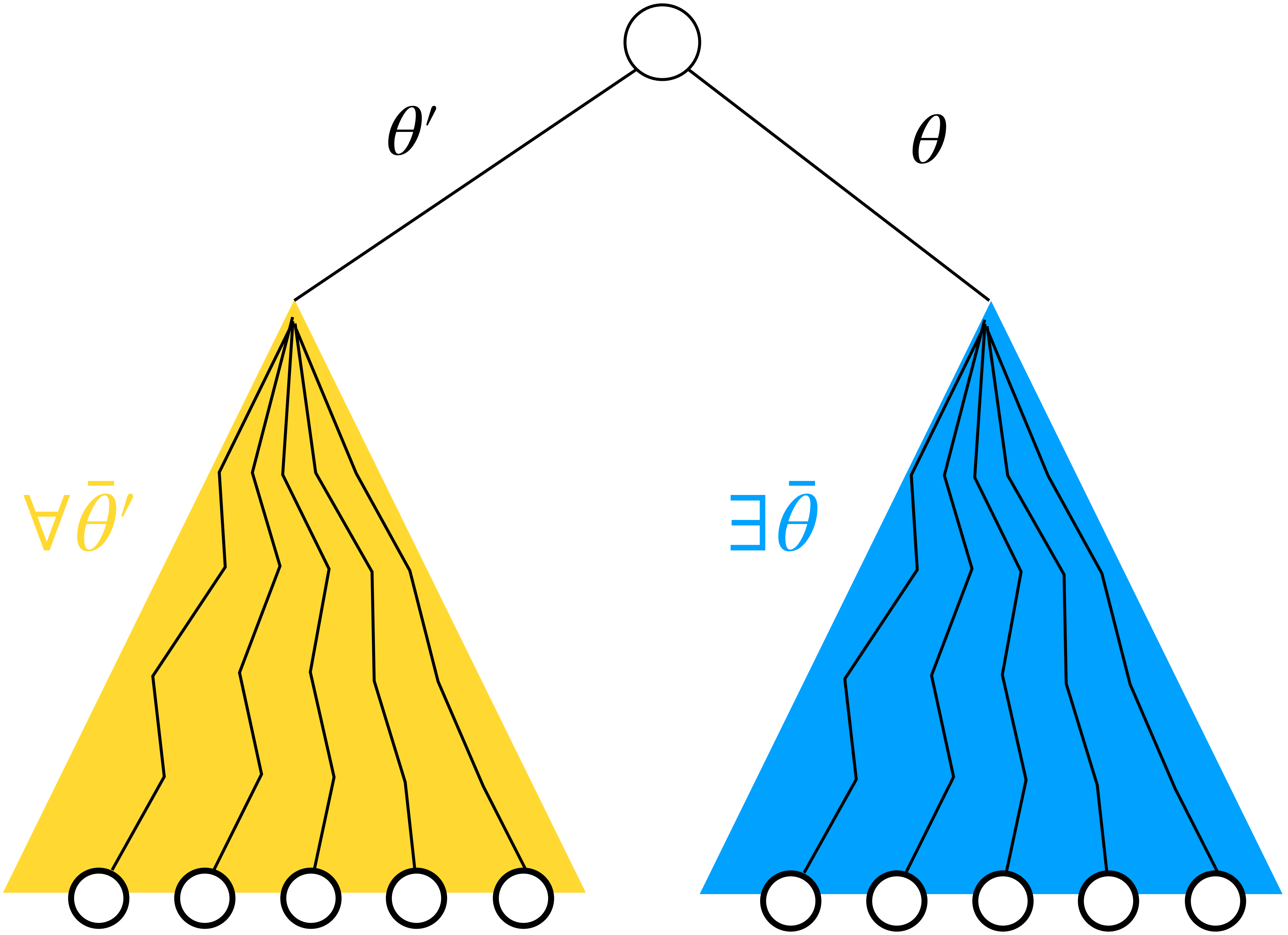
$\exists\bar{\theta}$ extending θ

s.t. $\bar{\theta} \prec \bar{\theta}'$

Theorem:
if θ dominates θ', we can add $\neg\theta'$ to COP

Automatic Dominance Breaking

(Lee and Zhong 2020)



Generation Problems:

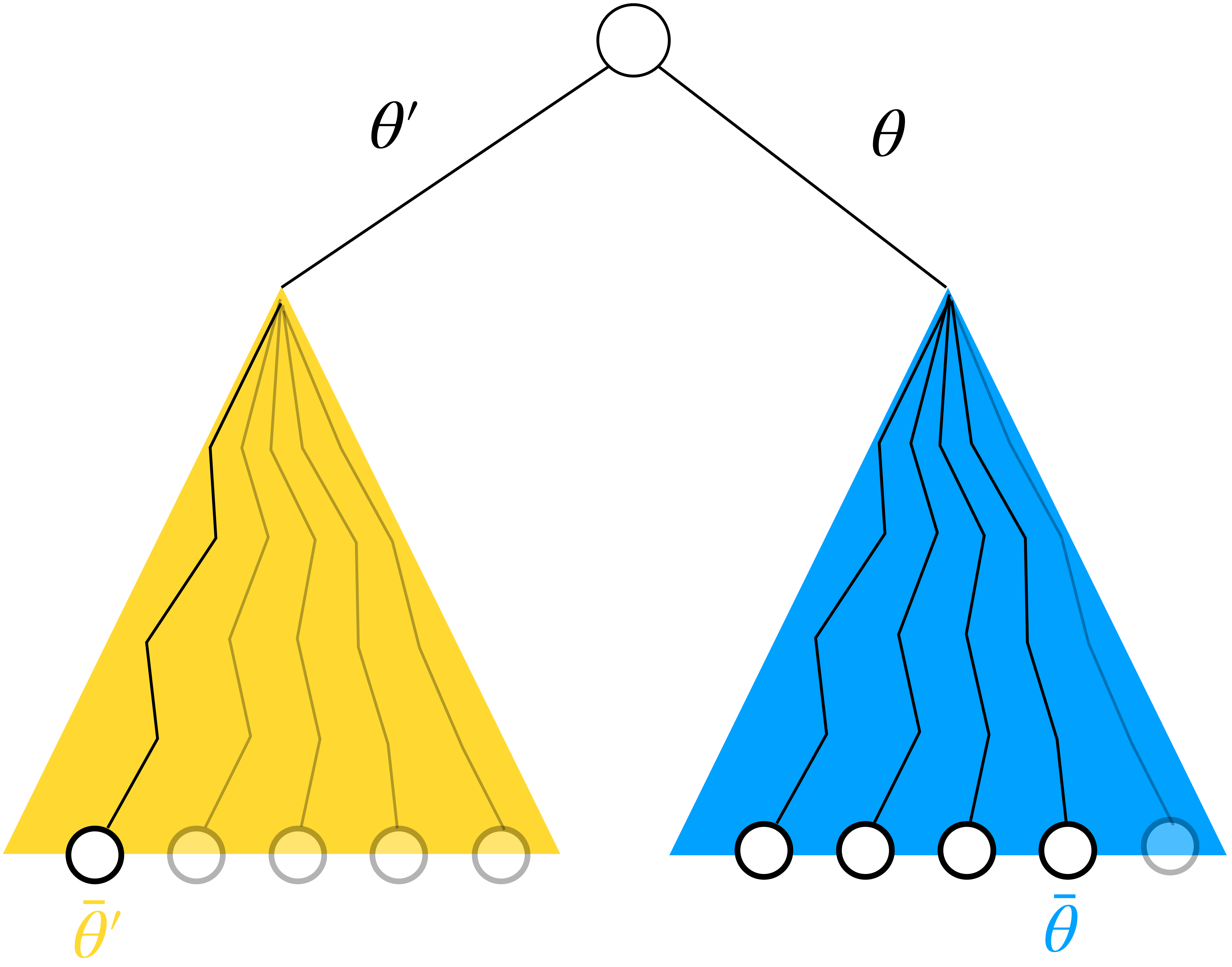
find pairs (θ, θ') such that:

(1) $\theta \prec \theta'$

Want to show $\forall\bar{\theta}'\ \exists\bar{\theta}$ s.t. $\bar{\theta} \prec \bar{\theta}'$

Automatic Dominance Breaking

(Lee and Zhong 2020)



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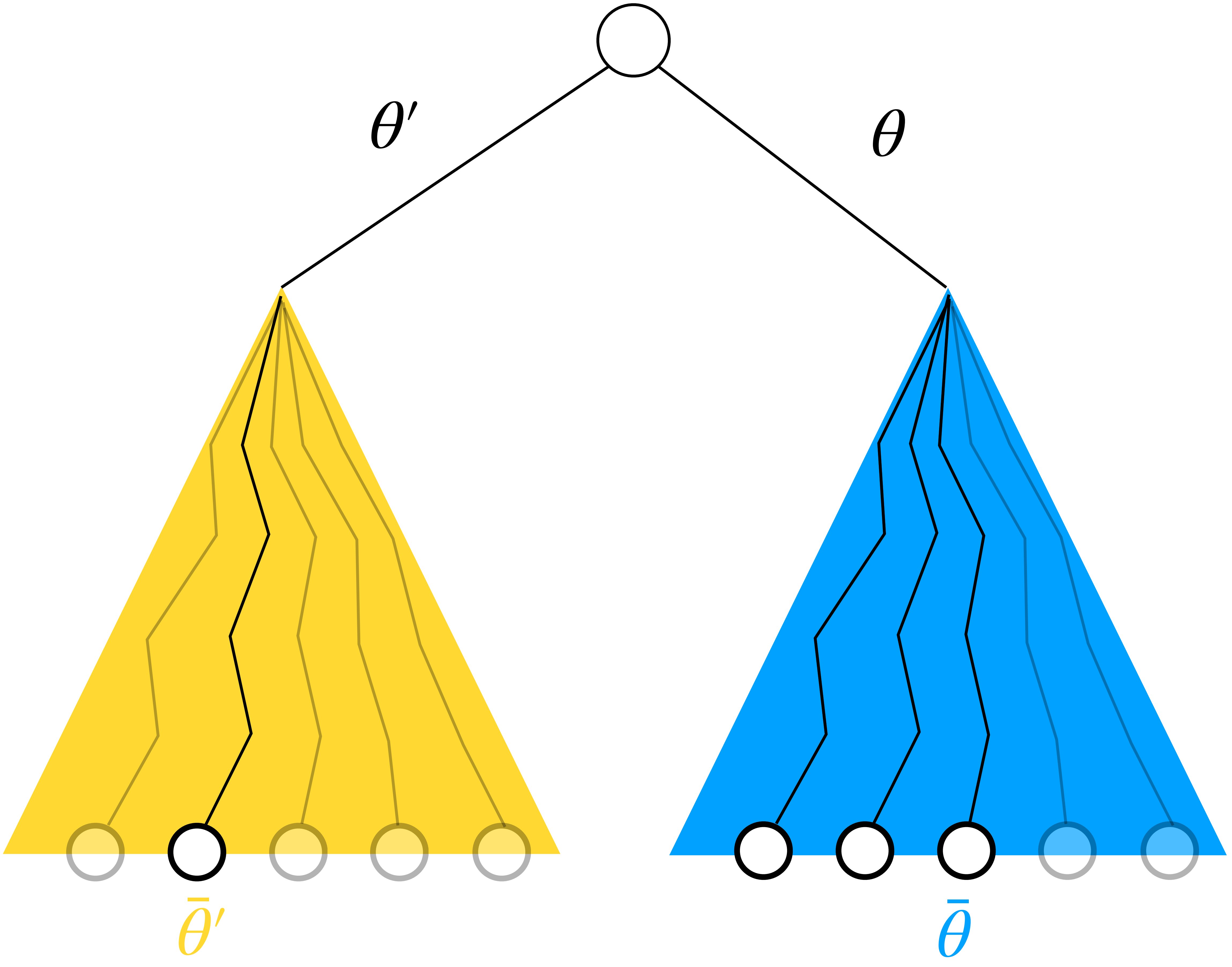

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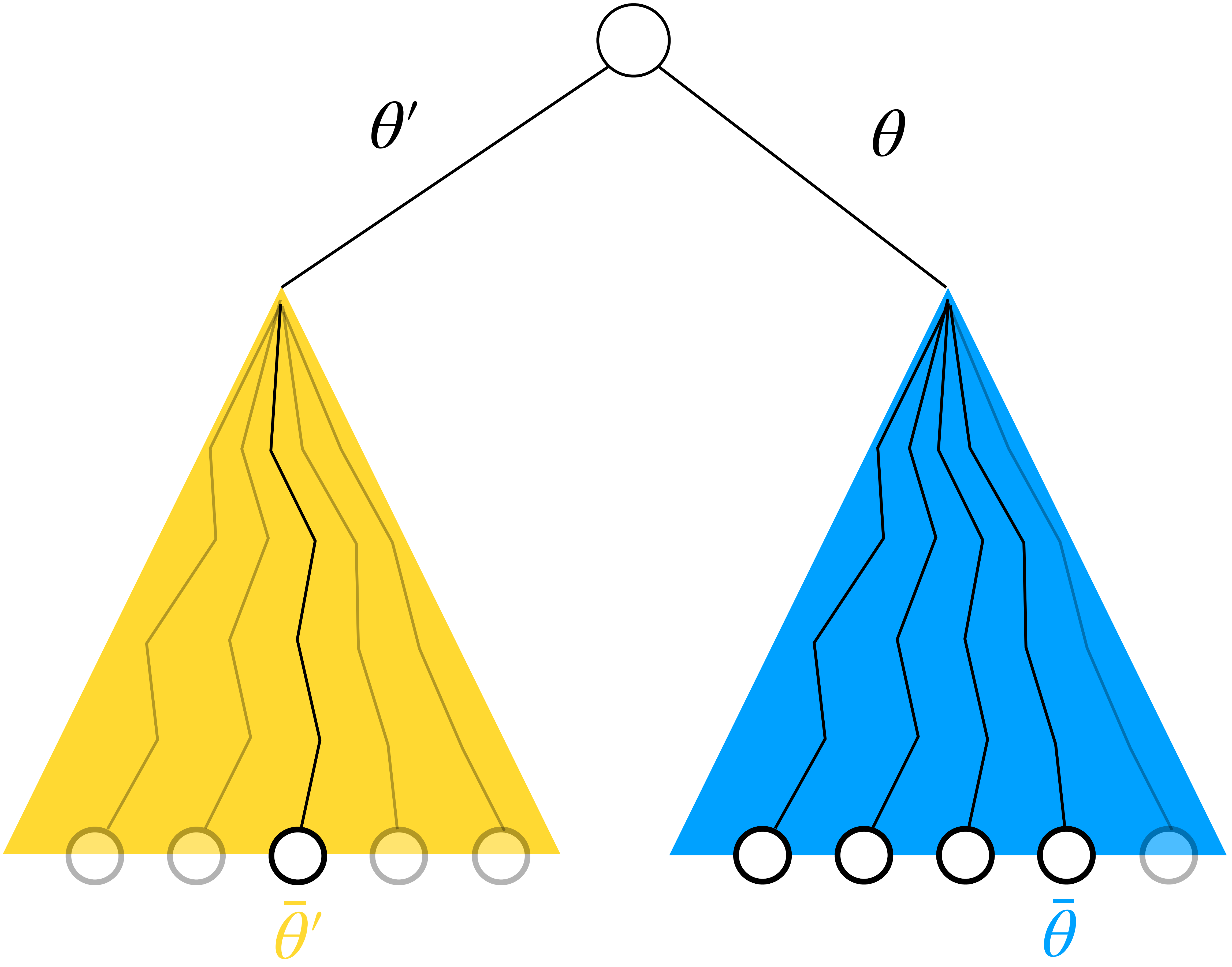

Generation Problems:

find pairs (θ, θ') such that:

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Automatic Dominance Breaking

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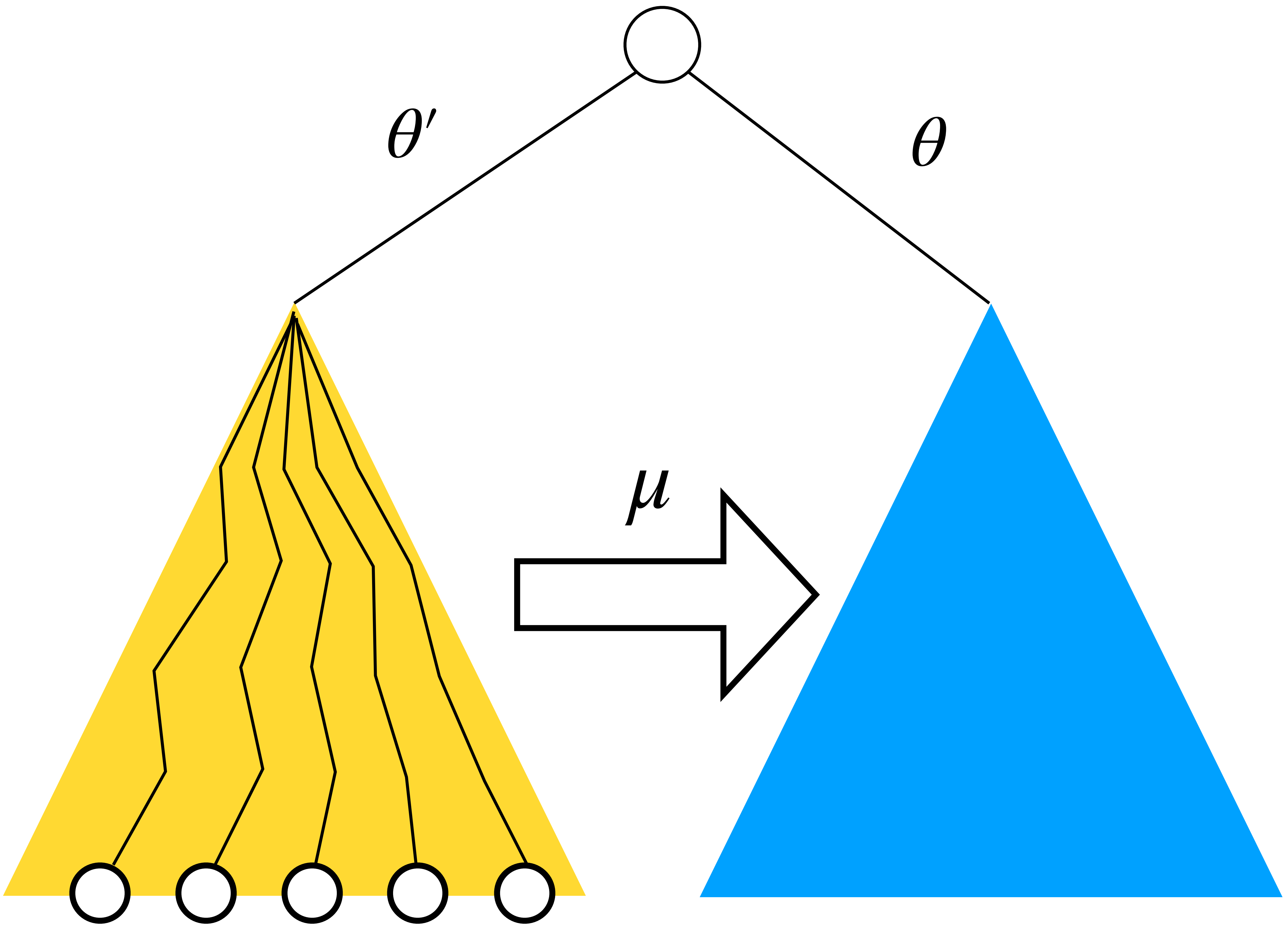


Generation Problems:

find pairs (θ, θ') such that:

(1) $\theta \prec \theta'$

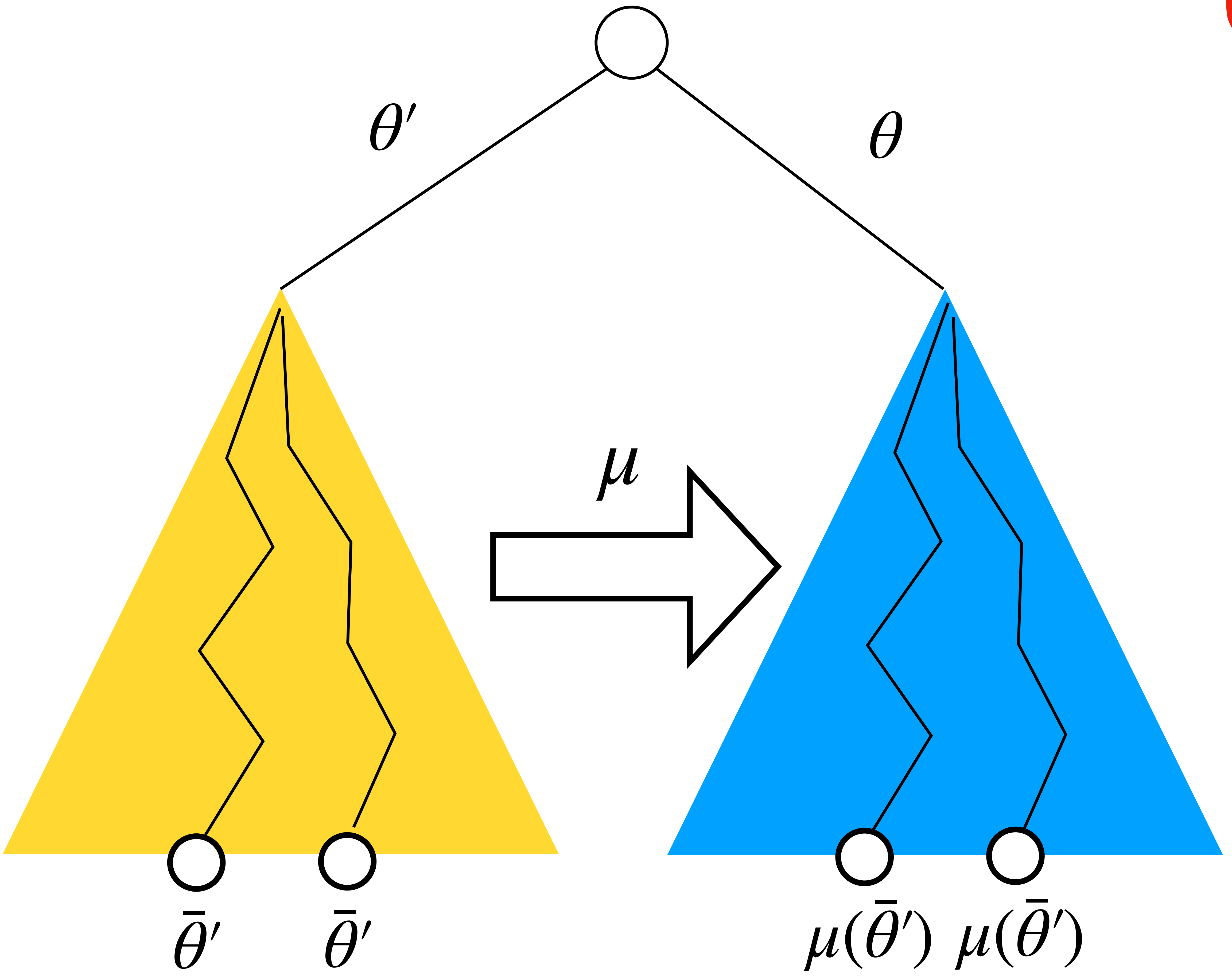
(2) $var(\theta) = var(\theta')$

Want to show $\forall \bar{\theta}'\ \exists \bar{\theta}$ s.t. $\bar{\theta} \prec \bar{\theta}'$

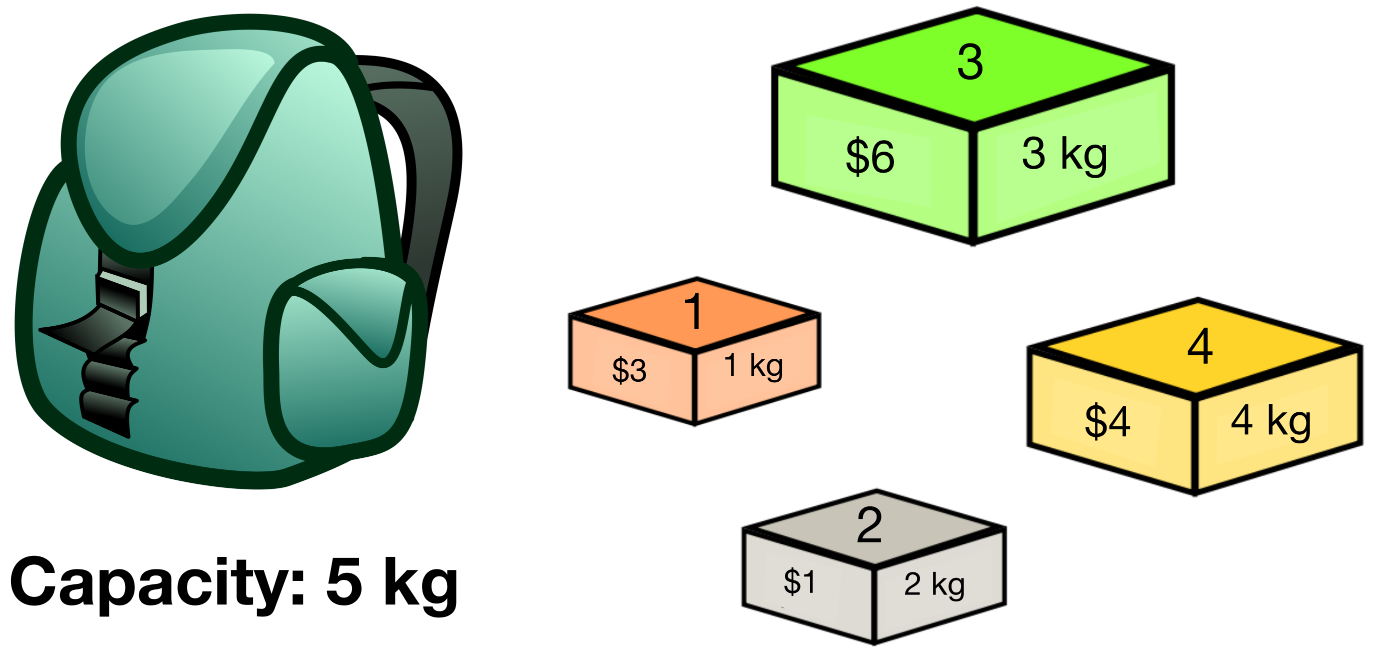
Suffice to show $\forall \bar{\theta}'$, $\mu(\bar{\theta}') \prec \bar{\theta}'$

Mutation Mapping

(Lee and Zhong 2020)



Mutation mapping: $\mu(\bar{\theta}')$ extends θ **in the same way** $\bar{\theta}'$ extends θ'



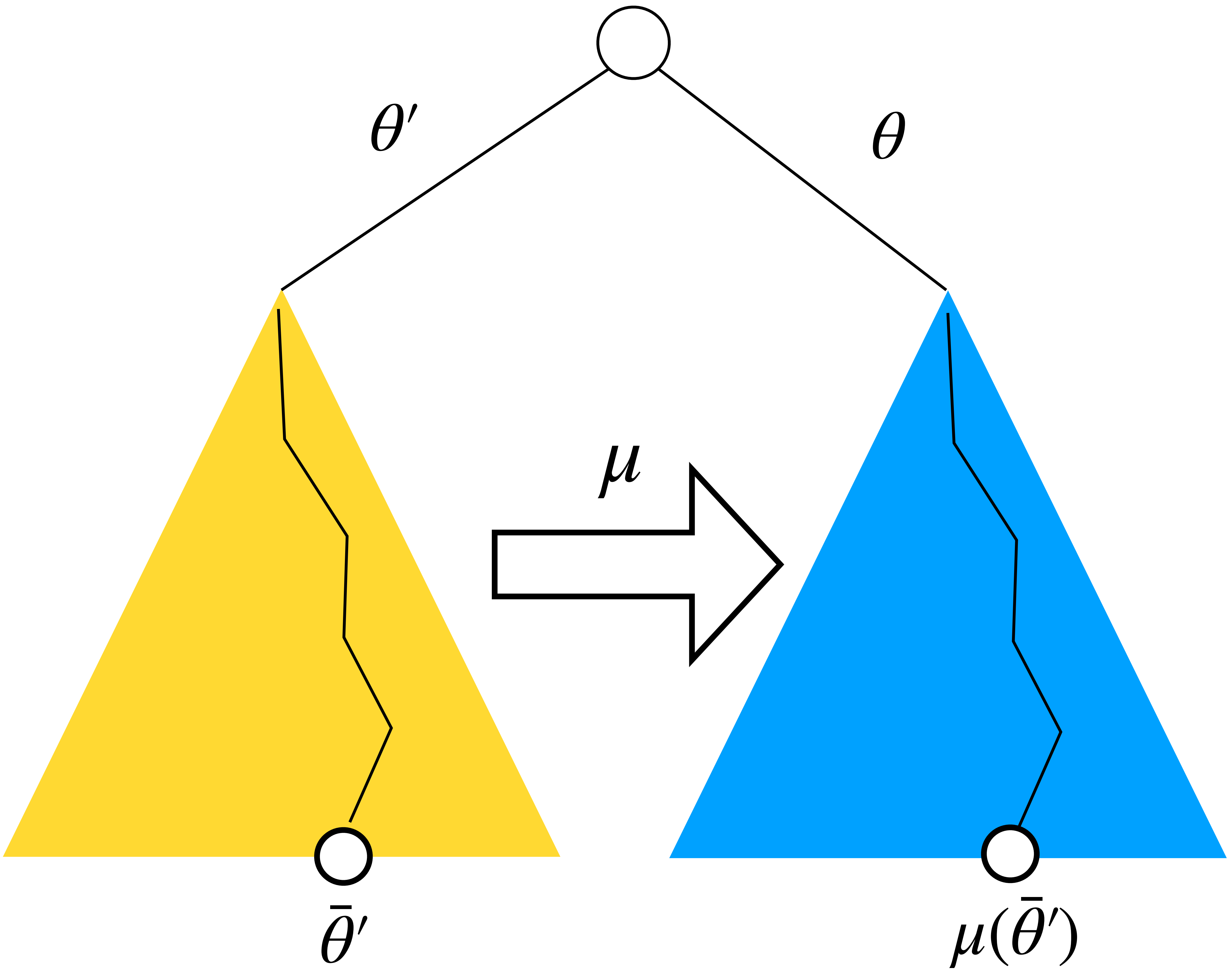
Example: Suppose $a, b, c, d \in \{0,1\}$

$\theta' = \{x_1 = a,\ x_2 = b\}$ $\qquad$ $\theta = \{x_1 = c,\ x_2 = d\}$

x1	x2	x3	x4	μ	x1	x2	x3	x4
a	b	0	0	$\mapsto$	c	d	0	0
a	b	0	1	$\mapsto$	c	d	0	1
a	b	1	0	$\mapsto$	c	d	1	0
a	b	1	1	$\mapsto$	c	d	1	1

Automatic Dominance Breaking

(Lee and Zhong 2020)



Mutation mapping: $\mu(\bar{\theta}')$ extends θ **<u>in the same way</u>** $\bar{\theta}'$ extends θ'

Theorem: $\theta \prec \theta'$ if

- Not equal: $\theta \neq \theta'$
- Betterment:
  $\forall \bar{\theta}', f(\mu(\bar{\theta}'))$ is better than $f(\bar{\theta}')$
- Implied satisfaction:
  $\forall \bar{\theta}', \bar{\theta}'$ solution $\Rightarrow \mu(\bar{\theta}')$ solution

Constraints over (θ, θ') !

Automatic Dominance Breaking

(Lee and Zhong 2020)

- Automatic dominance breaking is enabled for a class of COPs.

Efficiently Checkable Objectives	**Efficiently Checkable Constraints**
• Separable objectives • Submodular set objectives	• Domain constraints • Boolean disjunction constraints • Linear inequality constraints • Counting family constraints

Modelling

(Lee and Zhong 2020)

```
int: n;       % number of items
int: W;       % knapsack capacity
array [1..n] of int: w;  % weight of each item
array [1..n] of int: v;  % value of each item

array [1..n] of var 0..1: x;

% constraint
constraint sum (i in 1..n) (w[i]*x[i]) <= W;

% objective
solve maximize sum (i in 1..n) (v[i]*x[i]);
```

COP MiniZinc Model

```
int: n;       % number of items
int: W;       % knapsack capacity
array [1..n] of int: w;  % weight of each item
array [1..n] of int: v;  % value of each item
int: k; % size of partial assignments
```

Generation CSP MiniZinc Model

Modelling

(Lee and Zhong 2020)

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int: n;       % number of items
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array [1..n] of int: w;  % weight of each item
array [1..n] of int: v;  % value of each item
int: k; % size of partial assignments

array [1..k] of var 1..n: F; % indices for fixed variable
array [1..k] of var 0..1: v1; % fixed value for \theta
array [1..k] of var 0..1: v2; % fixed value for \theta'
constraint increasing(F); % symmetry breaking
constraint lex_less(v1,v2); % compatibility and not equal
```

Generation CSP MiniZinc Model

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(Lee and Zhong 2020)

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COP MiniZinc Model

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int: n;         % number of items
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array [1..k] of var 1..n: F; % indices for fixed variable
array [1..k] of var 0..1: v1; % fixed value for \theta
array [1..k] of var 0..1: v2; % fixed value for \theta'
constraint increasing(F); % symmetry breaking
constraint lex_less(v1,v2); % compatibility and not equal

% constraint for implied satisfaction
constraint sum(t in 1..k)( w[F[t]] * v1[t] )
           <=  sum(t in 1..k)( w[F[t]] * v2[t] );
```

Generation CSP MiniZinc Model

Modelling

(Lee and Zhong 2020)

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int: n;        % number of items
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COP MiniZinc Model

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int: n;        % number of items
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array [1..k] of var 0..1: v2; % fixed value for \theta'
constraint increasing(F); % symmetry breaking
constraint lex_less(v1,v2); % compatibility and not equal

% constraint for implied satisfaction
constraint sum(t in 1..k)( w[F[t]] * v1[t] )
             <=  sum(t in 1..k)( w[F[t]] * v2[t] );

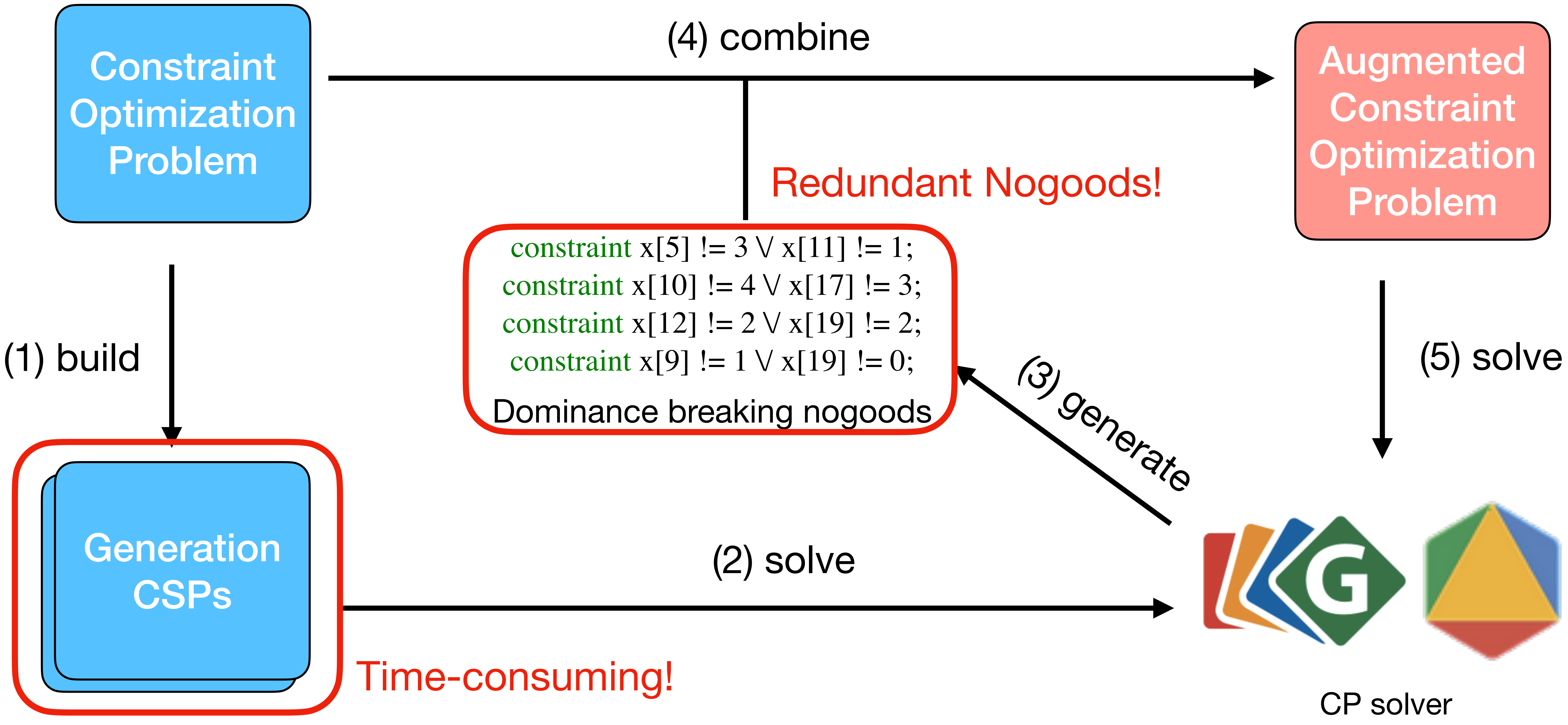
% constraint for betterment
constraint sum(t in 1..k)( v[F[t]] * v1[t] )
             >= sum(t in 1..k)( v[F[t]] * v2[t] );
```

Generation CSP MiniZinc Model

Common Assignment Elimination

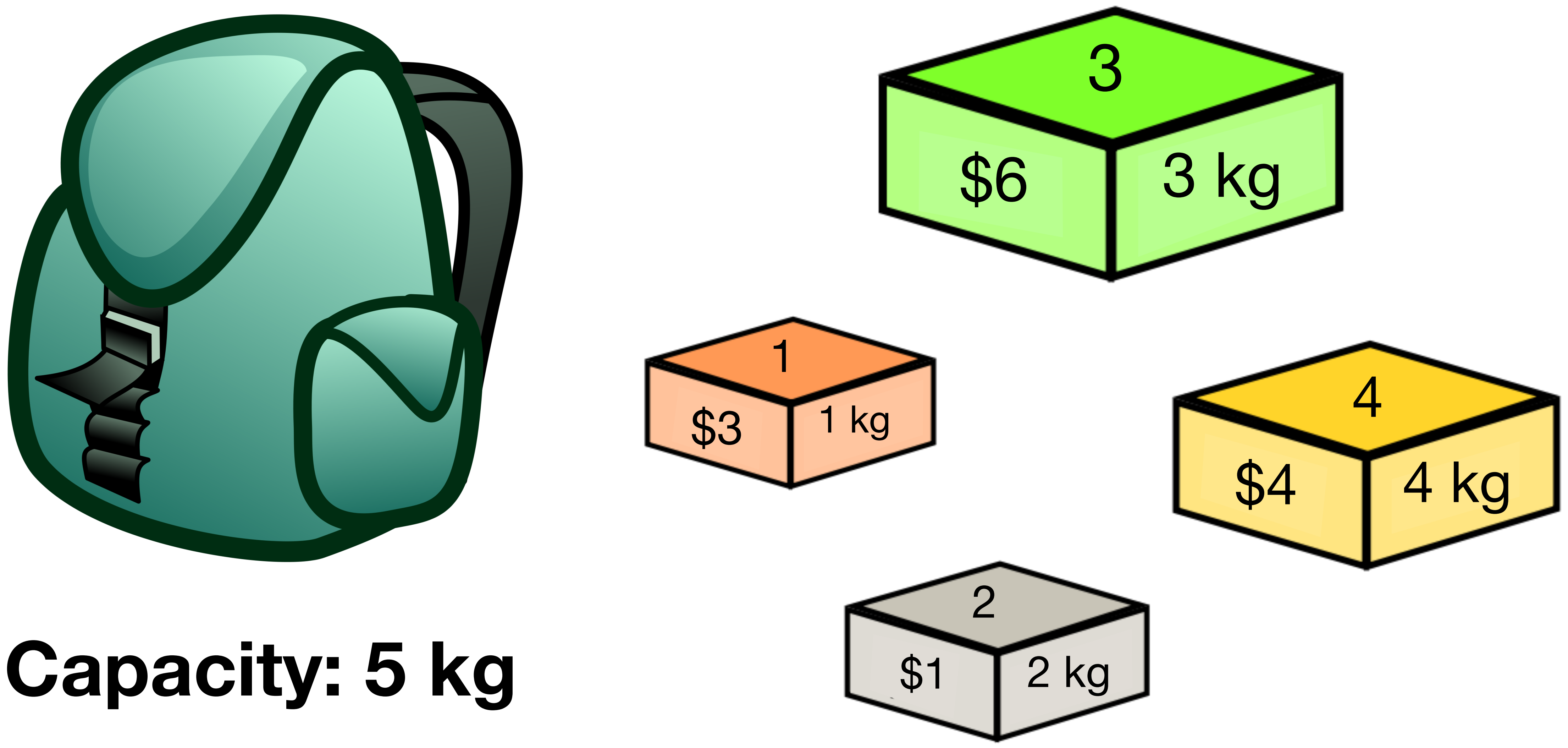
(Lee and Zhong 2021)

- Automatic dominance breaking is not efficient enough.



Common Assignment Elimination

(Lee and Zhong 2021)



Capacity: 5 kg

maximize $x_1 + 2x_2 + 4x_3 + 10x_4$

s.t. $x_1 + 2x_2 + 3x_3 + 4x_4 \leq 5$

$x_i \in \{0,1\}$ for $i = 1,\ldots,4$

$\cancel{c_1 \equiv (\boxed{x_2 \neq 1 \vee x_4 \neq 0} \vee x_3 \neq 1)}$

$c_2 \equiv (\boxed{x_2 \neq 1 \vee x_4 \neq 0})$

Avoid generating c_1 by common assignment elimination

$c_2 \Rightarrow c_1$

c_1 is also propagation redundant!

Exploiting Functional Constraints

(Lee and Zhong 2022)

- Automatic dominance breaking is enabled only for a class of COPs.

Efficiently Checkable Objectives	Efficiently Checkable Constraints
• Separable objectives • Submodular set objectives	• Domain constraints • Boolean disjunction constraints • Linear inequality constraints • Counting family constraints

- Impractical restriction on efficiently checkable objectives and constraints.

Exploiting Functional Constraints

(Lee and Zhong 2022)

- Automatic dominance breaking is enabled only for a class of COPs.
- Constraint programming provides a flexible modelling language which can form various objectives and constraints.
  - Example:

$$\text{minimize } \boxed{\max(x_1, x_2)} + 4x_3$$

$$\text{s.t. } 2x_1 - \boxed{3x_2 * x_3} \leq 5$$

$$x_i \in \{0,1\} \text{ for } i = 1,2,3$$

Not efficiently checkable!

- Unknown constraints for betterment and implied satisfaction in generation CSPs

Exploiting Functional Constraints

(Lee and Zhong 2022)

- COPs specified in a modelling language are normalised/flattened into a form with only standard constraints from the underlying solver.
  - Example:

minimize $\max(x_1, x_2) + 4x_3$

s.t. $2x_1 - 3x_2 * x_3 \leq 5$

$x_i \in \{0,1\}$ for $i = 1,2,3$

normalised $\longrightarrow$

Functional constraints

minimize obj

s.t. $\boxed{obj = y_1 + 4x_3,\ y_1 = \max(x_1, x_2)}$

$y_2 \leq 5, \boxed{y_2 = 2x_1 - 3y_3,\ y_3 = x_2 * x_3}$

$x_i \in \{0,1\}$ for $i = 1,2,3$

Auxiliary variables $\boxed{y_1, y_2, y_3, obj \in \mathbb{Z}}$

- Motivation: exploit standard functional constraints from CP solvers

Experimental Settings

- Modify the compiler for the MiniZinc modelling language
  - Available online: https://github.com/AllenZzw/auto-dom
- Experiment on talent scheduling, maximum coverage, sensor placement
  - Chuffed for problem-solving, Geas for nogood generation
  - 20 random instances for each problem size
  - 2 hours total timeout; reserve 1 hour for nogood generation.

Experimental Evaluation

- Geometric mean of time (seconds) for problem of different sizes:

Problem	Basic	2-dom		3-dom		4-dom	
		Solving	Total	Solving	Total	Solving	Total
Team-6-5	24.48	10.57	**12.49**	9.70	32.00	8.88	427.73
Team-7-5	276.84	138.88	**146.15**	130.71	225.19	150.83	1745.96
Team-8-5	1983.53	819.58	**829.05**	767.52	1024.43	724.63	5191.70
MaxCover-45	75.91	53.47	53.79	5.07	**9.96**	0.27	83.93
MaxCover-50	615.04	464.81	465.53	26.31	**34.92**	1.12	134.99
MaxCover-55	3576.98	2859.60	2860.27	78.37	**91.53**	2.54	199.11
Sensor-50	156.84	138.65	139.44	94.05	**108.99**	57.34	297.18
Sensor-60	595.46	404.27	405.52	269.61	**296.56**	172.43	709.37
Sensor-70	1615.18	1144.17	1145.83	810.01	**854.61**	651.72	1724.70

Concluding Remarks

- Automatic dominance breaking
  - Generating dominance breaking nogoods as constraint satisfaction
  - Automatically derive sufficient conditions in generation CSPs
- Future work
  - Nogood generation from constraint models alone
  - Dynamic generation of dominance breaking nogoods

Thanks!

Experimental Evaluation

- Geometric mean of time (seconds) for problem of different sizes:

Problem	Basic	Manual	2-dom		3-dom		4-dom	
			Solving	Total	Solving	Total	Solving	Total
Talent-16	187.79	5929.75	189.95	192.16	130.78	**148.91**	256.46	1988.75
Talent-18	1575.51	7200.0	1565.89	1568.29	672.26	**713.55**	1864.68	5760.68
Talent-20	5013.10	7200.0	4936.18	4960.54	2856.33	**2960.09**	3268.72	7006.10
Warehouse-35	7200.0	N/A	10.29	**52.11**	8.53	2442.71	8.51	3619.87
Warehouse-40	7200.0	N/A	46.08	**111.43**	32.93	3652.15	32.55	3657.33
Warehouse-45	7200.0	N/A	69.41	**140.92**	45.45	3690.84	46.19	3694.63
Team-6-5	24.48	N/A	10.57	**12.49**	9.70	32.00	8.88	427.73
Team-7-5	276.84	N/A	138.88	**146.15**	130.71	225.19	150.83	1745.96
Team-8-5	1983.53	N/A	819.58	**829.05**	767.52	1024.43	724.63	5191.70

Experimental Evaluation

- Geometric mean of time (seconds) for problem of different sizes:

Problem	Basic	Manual	2-dom		3-dom		4-dom	
			Solving	Total	Solving	Total	Solving	Total
MaxCover-45	75.91	N/A	53.47	53.79	5.07	**9.96**	0.27	83.93
MaxCover-50	615.04	N/A	464.81	465.53	26.31	**34.92**	1.12	134.99
MaxCover-55	3576.98	N/A	2859.60	2860.27	78.37	**91.53**	2.54	199.11
PartialCover-45	2383.2	N/A	366.17	368.03	59.44	**70.64**	2.49	90.25
PartialCover-50	3769.26	N/A	780.80	781.73	74.86	**88.45**	6.86	153.90
PartialCover-55	4640.06	N/A	1769.31	1770.42	211.83	**234.41**	15.23	240.68
Sensor-50	156.84	N/A	138.65	139.44	94.05	**108.99**	57.34	297.18
Sensor-60	595.46	N/A	404.27	405.52	269.61	**296.56**	172.43	709.37
Sensor-70	1615.18	N/A	1144.17	1145.83	810.01	**854.61**	651.72	1724.70