Efficiently Explaining CSPs with Unsatisfiable Subset Optimization

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ModRef 2022

Examples of Constraint Satisfaction Problems

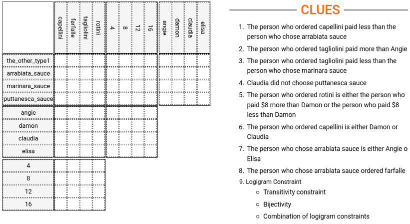


Figure: Logic Grid Puzzle

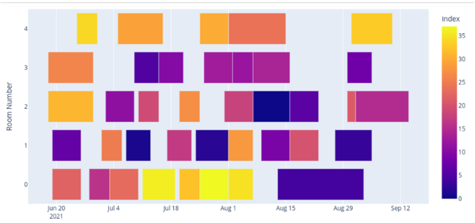


Figure: (Room) Scheduling Problem

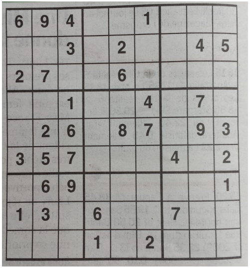


Figure: Sudoku

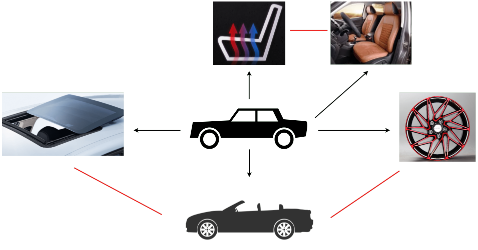


Figure: (Car) configuration problems

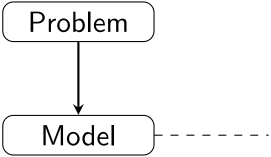
Constraint Satisfaction Problems

Solving

Problem

Constraint Satisfaction Problems

Solving



CPMpy (Guns [2019])
(CP modelling Library)
`pip install cpmpy`

Constraint Satisfaction Problems

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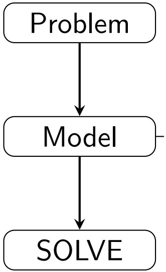

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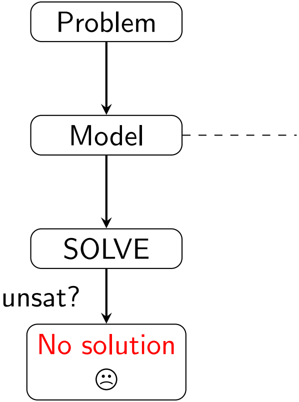

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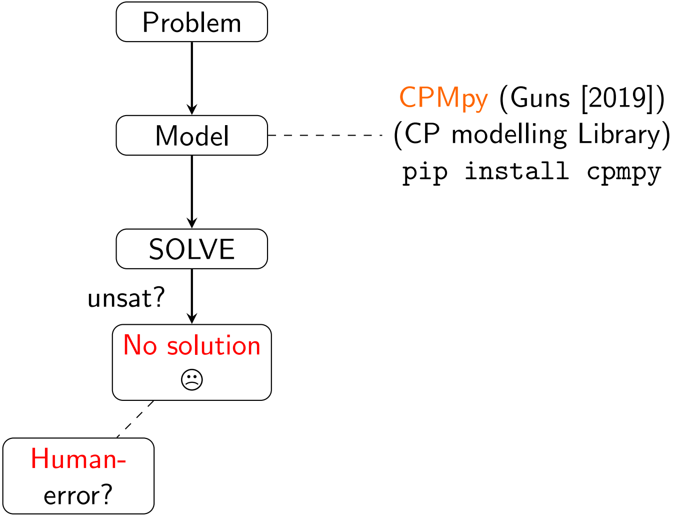

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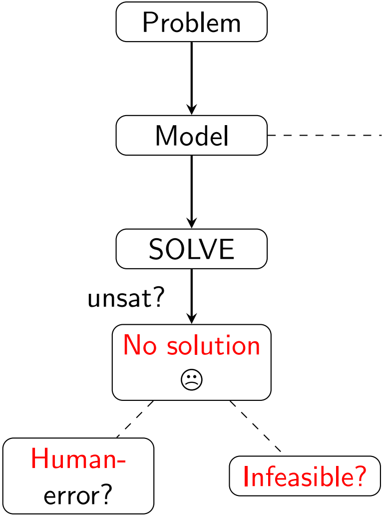

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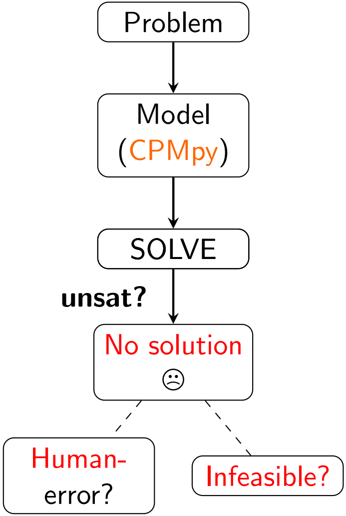
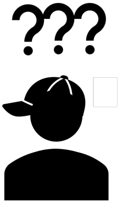
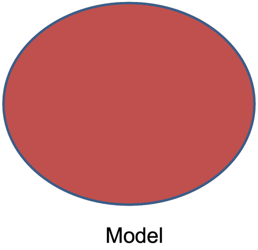


How do you explain UNSAT ?



How do I debug my model

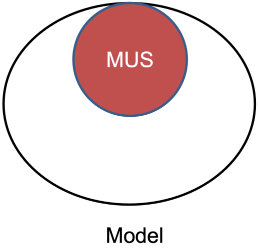
Explanations of unsatisfiability



How do I debug my model

Explanations of unsatisfiability

1. Identify conflicting constraints as an explanation (Liffiton and Sakallah [2008]; Ignatiev *et al.* [2015]...)
   → Extract a **Minimum Unsatisfiable Subset** (**MUS**)
   = Irreducible Inconsistent Subsystem (ISS)



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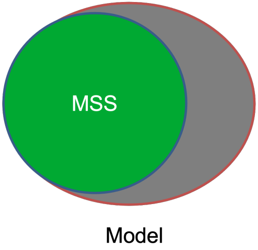
2. Identify a **Maximal Satisfiable Subset** (**MSS**) (Ignatiev *et al.* [2019]; Davies and Bacchus [2013]; Hansen and Jaumard [1990]...)



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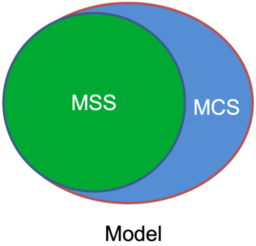
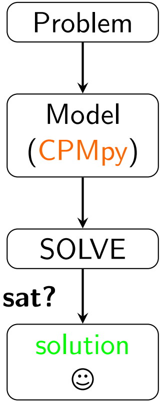
2. Identify a **Maximal Satisfiable Subset** (**MSS**) (Ignatiev *et al.* [2019]; Davies and Bacchus [2013]; Hansen and Jaumard [1990]...)
3. "Correct" the infeasibility in the constraints (Liffiton and Malik [2013]...)
   - → Extract a **Minimum Correction Subset** (**MCS**)
   - = Complement of some **MSS**, removal/correction leads to a satisfiable subset



Motivation - Explaining SAT problems



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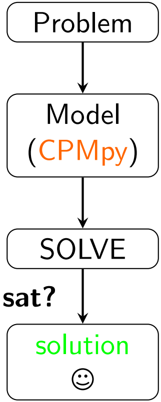

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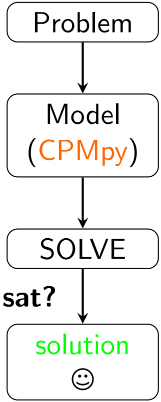

How do I explain SAT ?

- What is an **explanation**?

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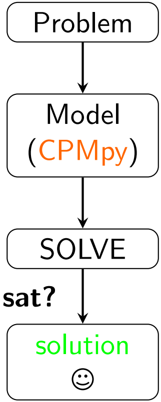

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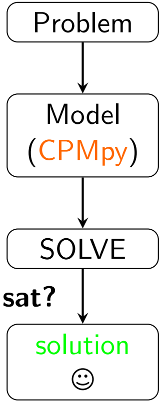


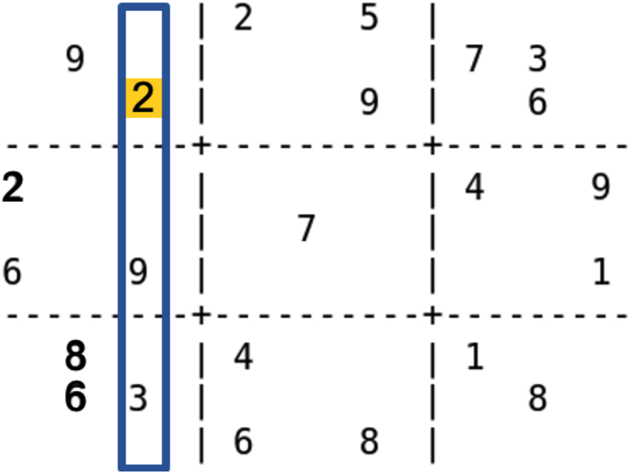
How do I explain SAT ?

- What is an **explanation**?
- What is a **good** explanation ?
- What is a **sequence of explanations** ?

What is an explanation ?

```
      |  2     5  |
   9  |           |  7  3
     2|        9  |     6
------+-----------+---------
2     |           |  4     9
      |     7     |
6    9|           |        1
------+-----------+---------
   8  |  4        |  1
   6 3|           |     8
      |  6     8  |
```

What is an explanation ?



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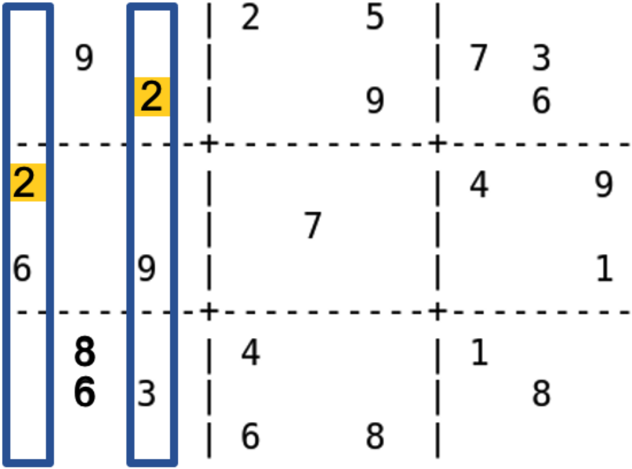

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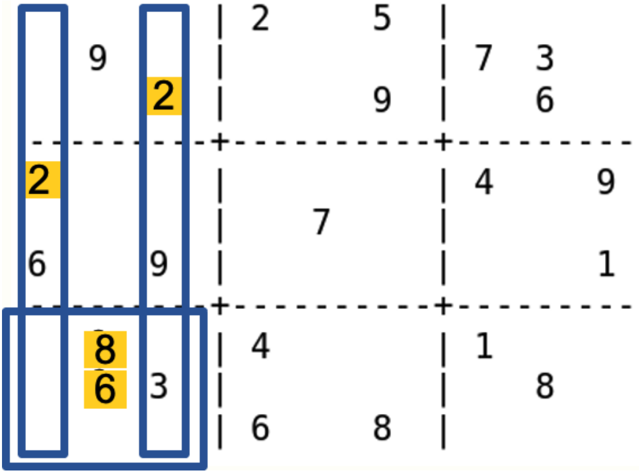

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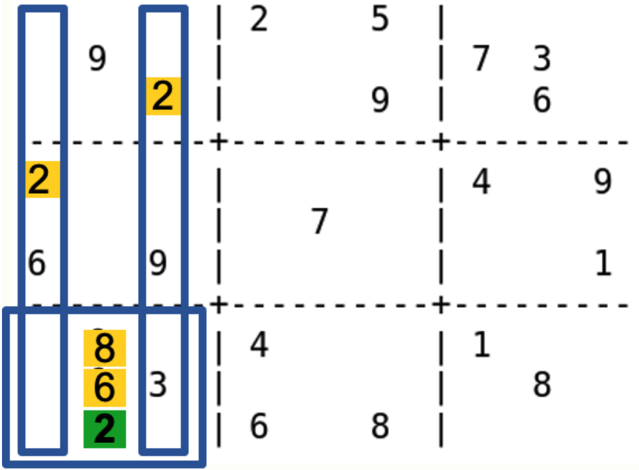

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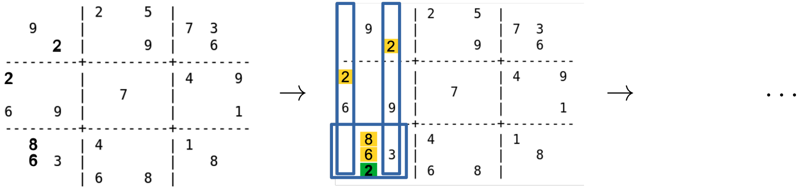

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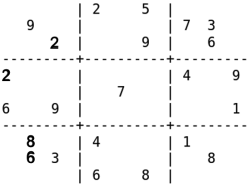
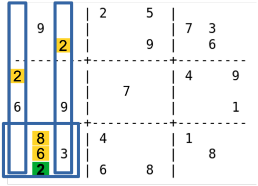
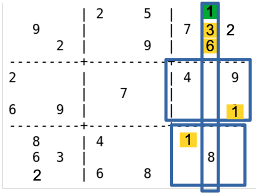


→

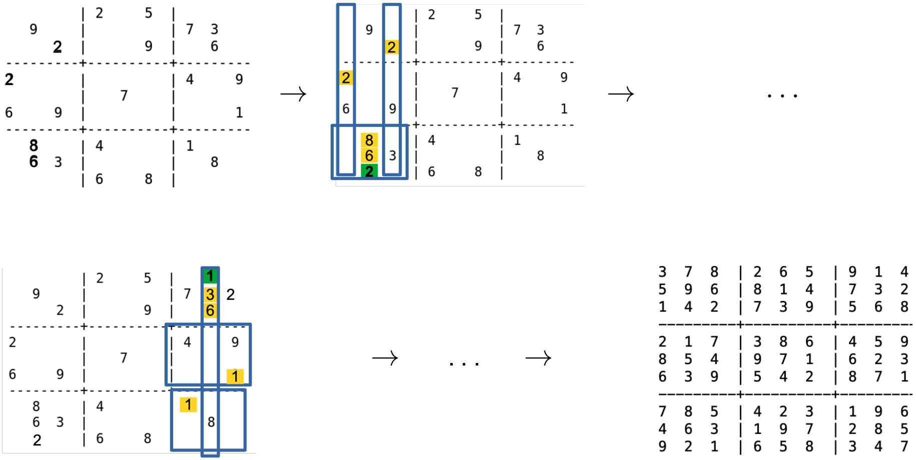


→ . . .



→ . . . →

What is a sequence of explanations ?



Stepwise explanations for CSPs

Goal

Given

C Constraints

E Facts, i.e. given sudoku numbers

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C Constraints

E Facts, i.e. given sudoku numbers

Goal:

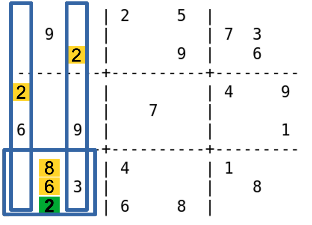
- Generate a *sequence* of *simple* explanations
- Explain step-by-step the solution of a Constraint Satisfaction Problem
- Explain 1 fact at a time

Stepwise explanations for CSPs

Explanation step - Formal definition

Let an EXPLANATION STEP (Bogaerts *et al.* [2020]) be:

$$E' \;\&\; C' \implies n$$



Stepwise explanations for CSPs

Explanation step - Formal definition

Let an EXPLANATION STEP (Bogaerts *et al.* [2020]) be:

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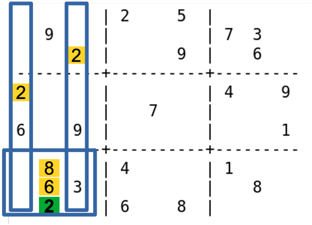
E' A subset of previously derived facts E

(Sudoku) Given and derived digits in the grid



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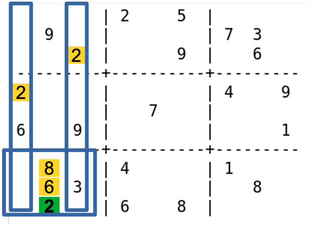

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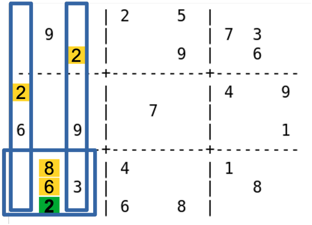

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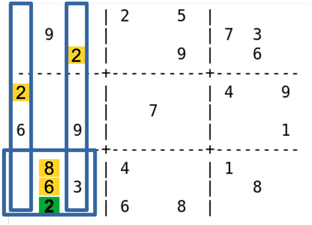

How ? MUS(E & C & ¬ n) is a valid explanation step

Stepwise explanations for CSPs

EXPLANATION STEP

Example

(Sudoku) Let E contain the assigned variables at the current state of the grid (e.g. $I = \{V_{(3,3)} = 2, \ldots\}$).

MUS(C & E &¬ ***n***)

$\{\texttt{alldifferent}(\{V_{(r,1)} | r \in 1..9\}), V_{(4,1)} = 2,$
$\texttt{alldifferent}(\{V_{(r,3)} | r \in 1..9\}), V_{(3,3)} = 2,$
$\texttt{alldifferent}(\{V_{(r_i,c_j)} | r_i \in 7..9, c_j \in 1..3\}),$
$V_{(7,2)} = 2, V_{(8,2)} = 2, V_{(9,2)} \neq 2\}$

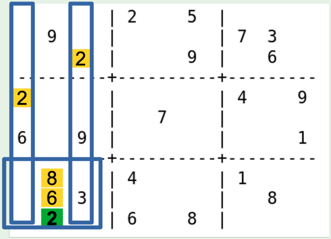

Figure: Example of a non-redundant explanation for $V_{(9,2)} = 2$

What is a good explanation ?

```
      |  2     5  |
   9  |           |  7  3
      2  |     9  |     6
---------+--------+---------
2        |        |  4     9
         |    7   |
6     9  |        |        1
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   8     |  4     |  1
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```

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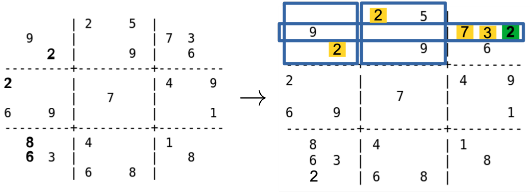

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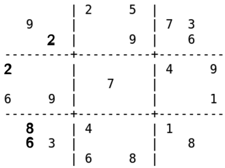
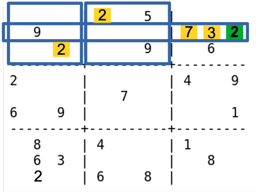


→



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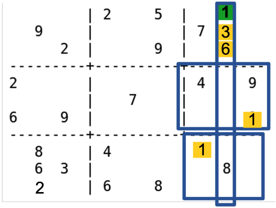

The best/easiest explanation step...

What is a good explanation ?

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What is a good explanation ?

Let **f**(E', C', ***n***) be a cost-function that quantifies how good (e.g. easy to understand) an explanation step is.

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What is the best/easiest explanation step?

$X_{best} \leftarrow nil$;

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$X_{best} \leftarrow nil$;

for ***n*** $\in$ `propagate(`C`)` **do**

  $X \leftarrow$ `MUS(` E & C &$\neg$ ***n*** `);`

The best/easiest explanation step...

What is a good explanation ?

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What is the best/easiest explanation step?

$X_{best} \leftarrow nil$;
for ***n*** $\in$ `propagate(`C`)` **do**
    $X \leftarrow$ `MUS(` E & C &$\neg$ ***n*** `)`;
    if $f(X) < f(X_{best})$ **then**
        $X_{best} \leftarrow X$;
end
return X_{best}

Motivation

Challenges and open questions from Bogaerts *et al.* [2020]

Q1 **Optimality** w.r.t **f**

- ▷ MUS guarantees non-redundancy but ... not optimality.
- ▷ Alternative: SMUS #-minimal (Ignatiev *et al.* [2015])

$X_{best} \leftarrow nil$;
for n $\in$ `propagate(`C`)` **do**
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Explanation generation takes a lot of time:

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Explanation generation takes a lot of time:

Q2 Can we avoid looping over the literals when searching for the next best explanation ?

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Explanation generation takes a lot of time:

Q2 Can we avoid looping over the literals when searching for the next best explanation ?

Q3 Can we reuse information from an explanation call to another?

```
X_best ← nil;
for n ∈ propagate(C) do
    X ← MUS(C ∧ E ∧ ¬n);
    if f(X) < f(X_best) then
        X_best ← X;
end
return X_best
```

OCUS-Based explanation generation

The OCUS[1] problem

Definition

Let $\mathcal{F}$ be a formula, $f : 2^{\mathcal{F}} \rightarrow \mathbb{N}$ a cost function and p a predicate $p : 2^{\mathcal{F}} \rightarrow \{\mathbf{t}, \mathbf{f}\}$. We call $\mathcal{S} \subseteq \mathcal{F}$ an OCUS of $\mathcal{F}$ (with respect to f and p) if

- $\mathcal{S}$ is unsatisfiable,
- $p(\mathcal{S})$ is true
- all other unsatisfiable $\mathcal{S}' \subseteq \mathcal{F}$ with $p(\mathcal{S}') = \mathbf{t}$ satisfy $f(\mathcal{S}') \geq f(\mathcal{S})$.

[1] Optimal Constrained Unsatisfiable Subset

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Applied to explanation generation:

[1] Optimal Constrained Unsatisfiable Subset

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The OCUS[1] problem

Definition

Let $\mathcal{F}$ be a formula, $f : 2^{\mathcal{F}} \rightarrow \mathbb{N}$ a cost function and p a predicate $p : 2^{\mathcal{F}} \rightarrow \{\mathbf{t}, \mathbf{f}\}$. We call $\mathcal{S} \subseteq \mathcal{F}$ an OCUS of $\mathcal{F}$ (with respect to f and p) if

- $\mathcal{S}$ is unsatisfiable,
- $p(\mathcal{S})$ is true
- all other unsatisfiable $\mathcal{S}' \subseteq \mathcal{F}$ with $p(\mathcal{S}') = \mathbf{t}$ satisfy $f(\mathcal{S}') \geq f(\mathcal{S})$.

Applied to explanation generation:

Q1 *(Optimality)* f ensures finding ‘best’ explanation

[1] Optimal Constrained Unsatisfiable Subset

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Applied to explanation generation:

Q1 *(Optimality)* f ensures finding 'best' explanation

Q2 *(Looping)* p allows formulating explaining 1 literal at a time using an extra constraint

[1] Optimal Constrained Unsatisfiable Subset

OCUS-Based explanation generation

$X_{best} \leftarrow nil$;
for $\mathtt{n} \in \mathtt{propagate}(C)$ **do**
    $X \leftarrow \mathrm{MUS}(C \wedge E \wedge \neg n)$;
    if $f(X) < f(X_{best})$ **then**
        $X_{best} \leftarrow X$;
end
return X_{best}

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$\downarrow$

$p \leftarrow$ `exactly-one`($\{\neg$ ***n*** $|$ ***n*** $\in DigitToExplain\}$)

`OCUS`(C $\wedge$ E $\wedge \{\neg$ ***n*** $|$ ***n*** $\in DigitToExplain\}$, **f**, **p**)

An OCUS algorithm: A hitting set-based algorithm

Implicit Hitting set duality

- Exploit implicit hitting set–based duality between MCSes and MUSes (Liffiton and Sakallah [2008]; Reiter [1987])

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  - Impose constraint on the hitting sets

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For example:

$$H_1 = \{c_3, c_5\}$$
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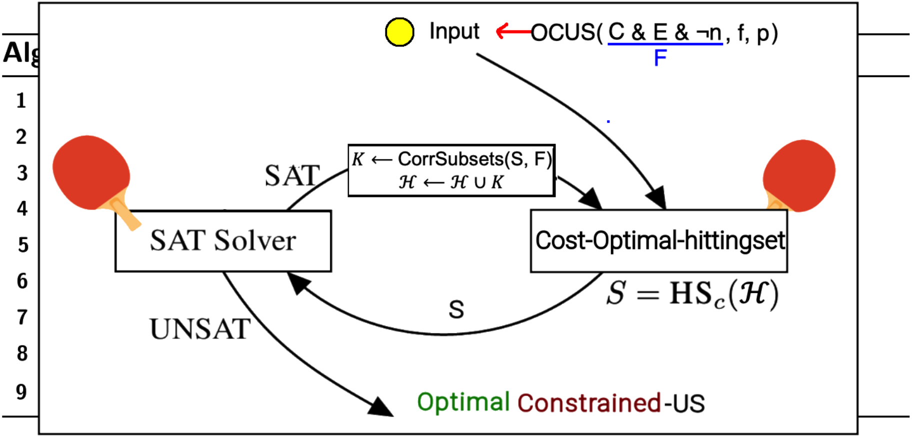
- $\{c_1, c_2, c_3\}$
  - cost: 7
- ...

OCUS: An efficient game of Ping-Pong



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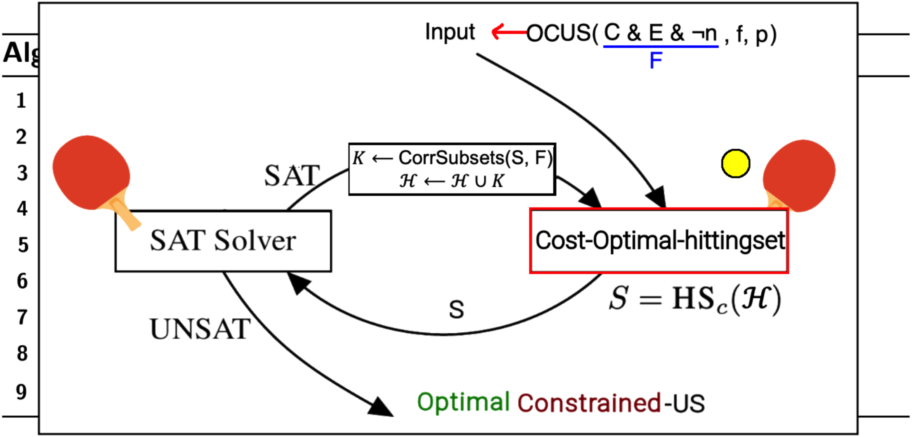

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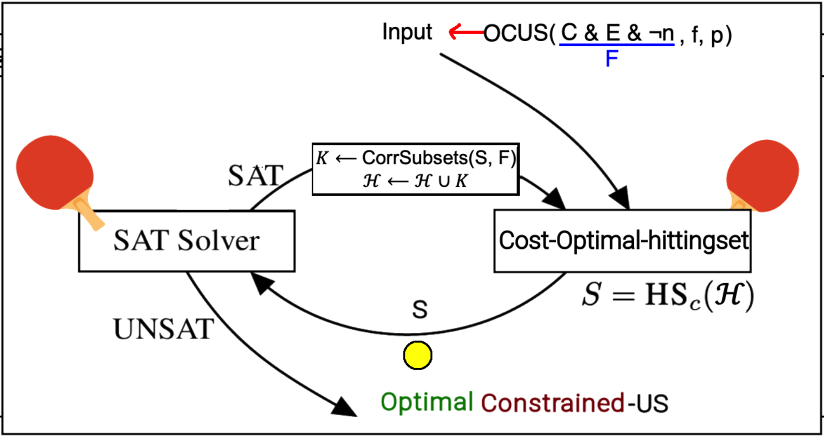

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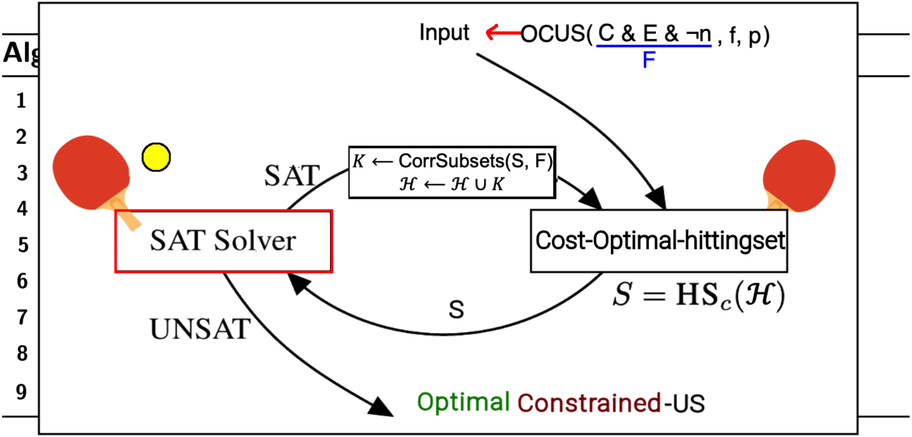

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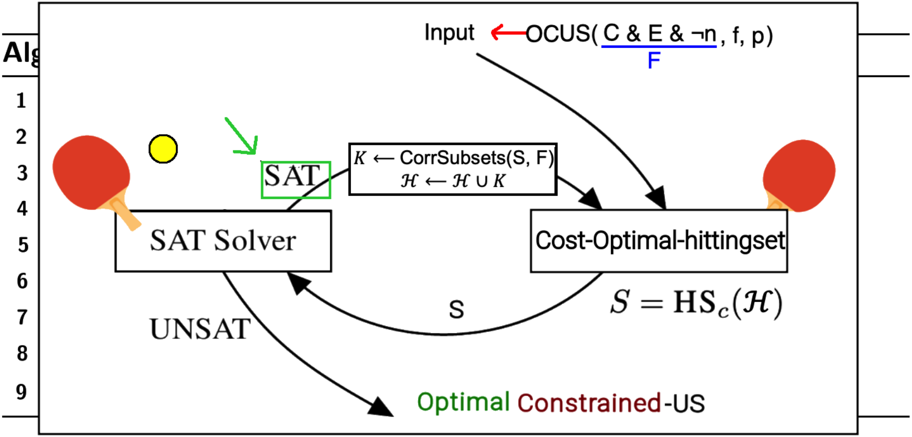
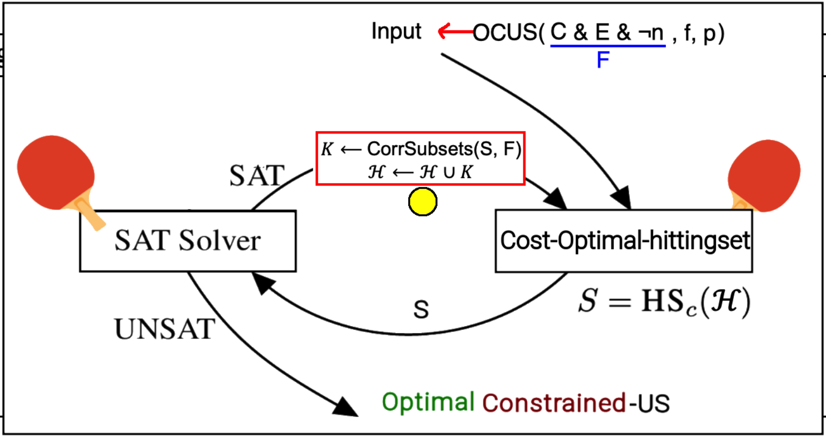


OCUS: An efficient game of Ping-Pong

Alg



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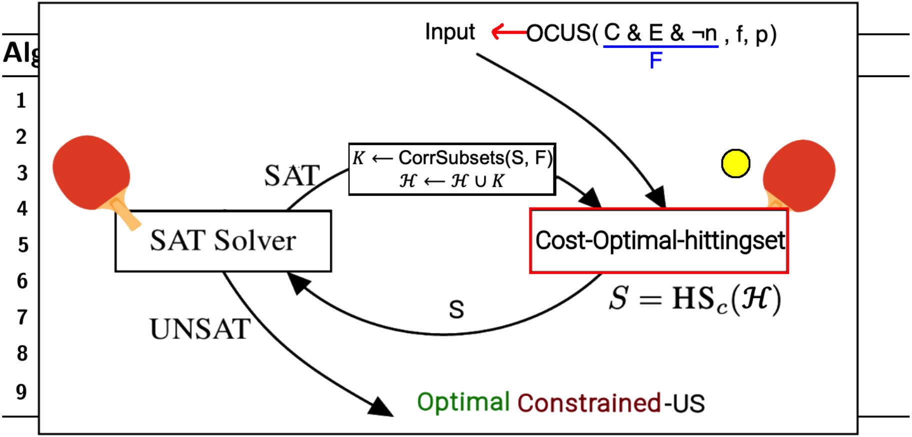

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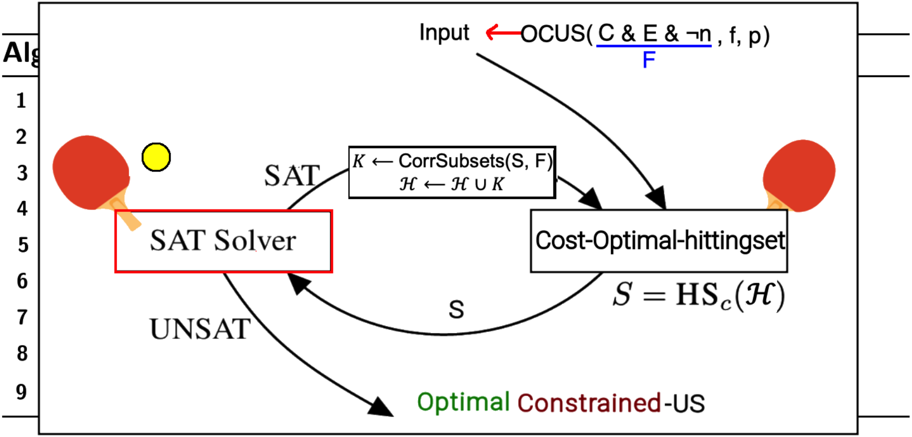

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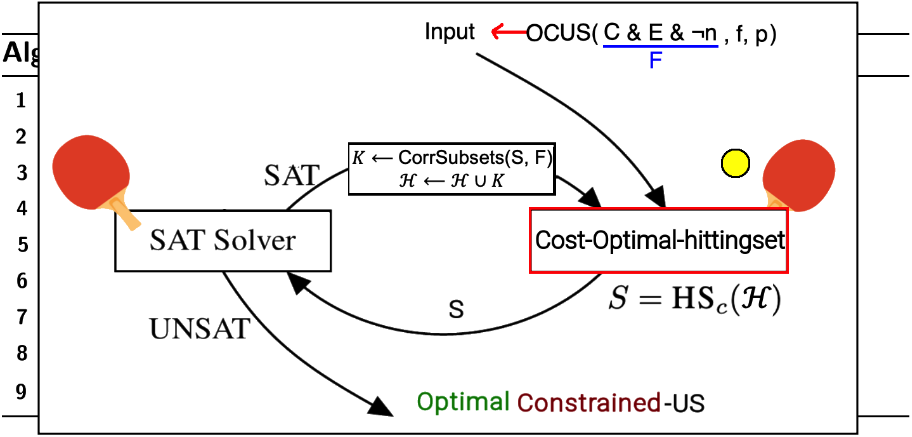

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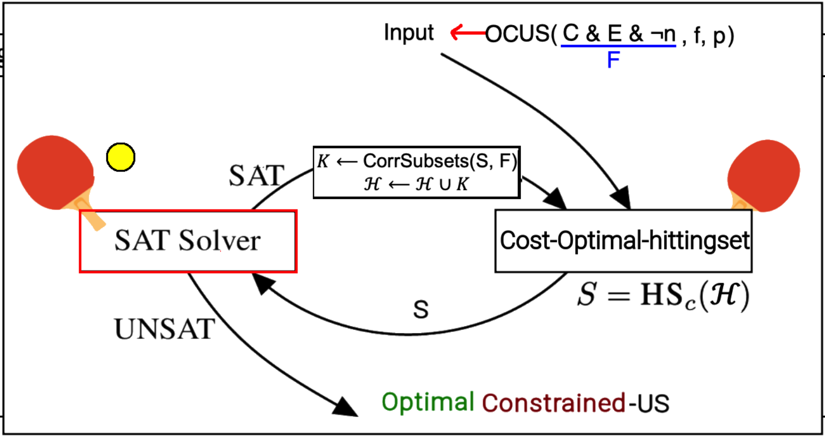

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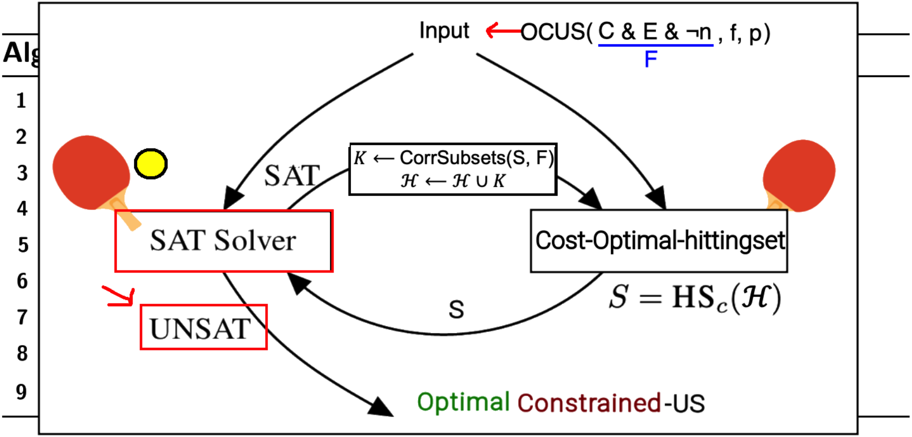

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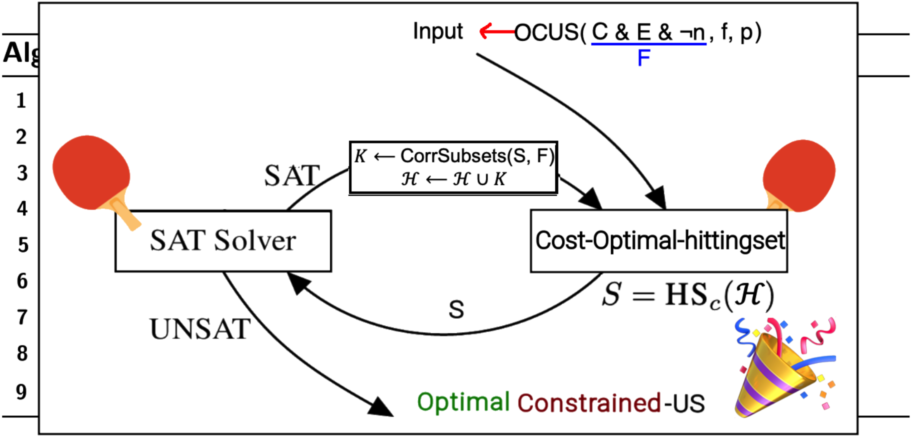

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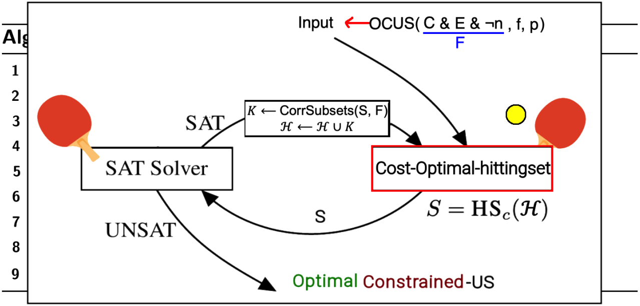
How do I efficiently compute OCUSs?



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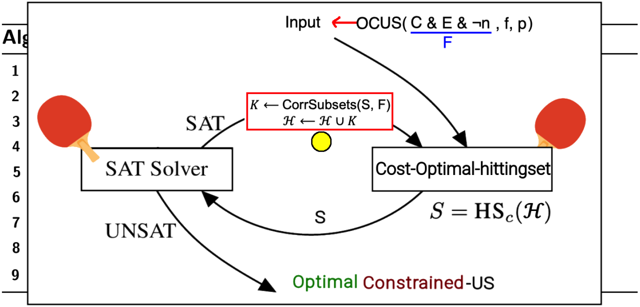

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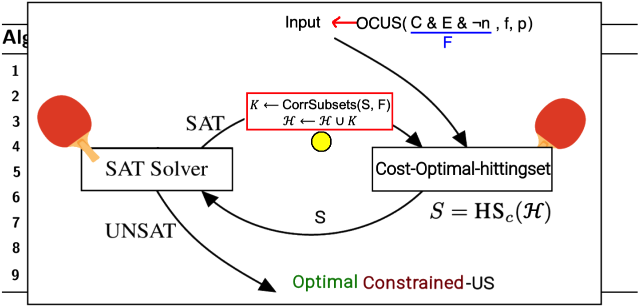

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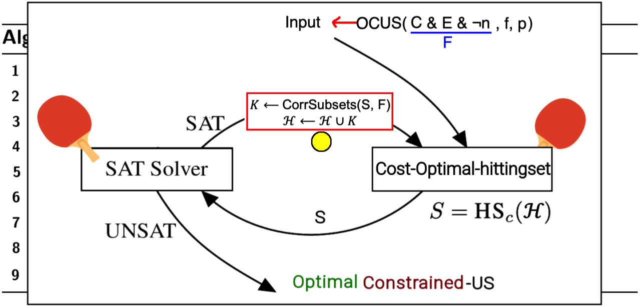

Correction-Subsets($\mathcal{S}, \mathcal{F}$)

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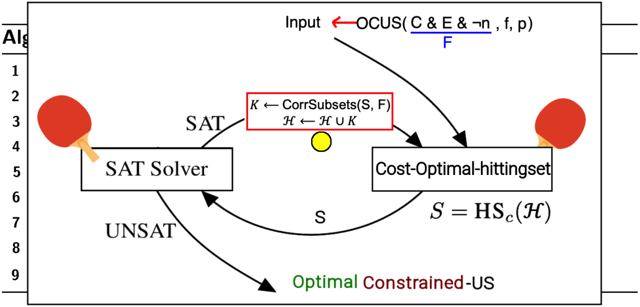

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  - $\rightarrow$ MaxSAT vs SAT-based heuristic
- Multiple *disjoint* correction subsets ?

Improving efficiency of the OCUS algorithm..

...in the context of explanation sequence generation

Can we improve OCUS-calls in the context of explanation sequence generation?

Improving efficiency of the OCUS algorithm..

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Incrementality

- Can we re-use of information between explanation calls ?

Improving efficiency of our OCUS algorithm..

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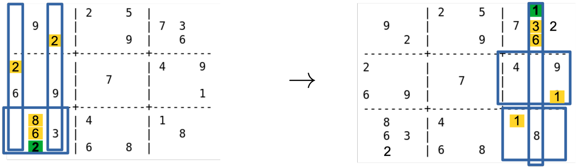
OCUS(C & E &¬ ***n*** , f, p)



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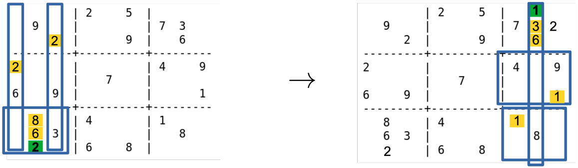

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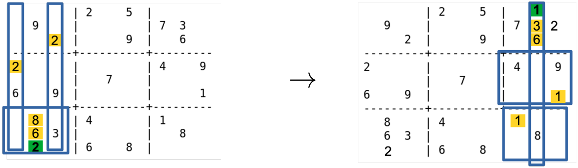

- C Model constraints
  - Do not change from an explanation step to another
- E Derived facts E

Improving efficiency of our OCUS algorithm..

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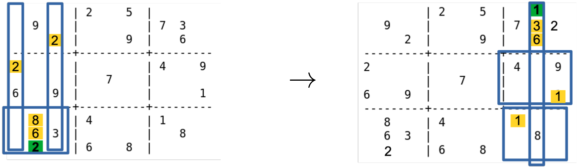
$$\textbf{OCUS}(\ \boxed{C}\ \&\ E\ \&\neg\ \boldsymbol{n}\ , f, p)$$



C Model constraints

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- Precision-increasing !

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Kick-start OCUS by bootstrapping $\mathcal{H}$

- Keep track of collection of correction sets $\mathcal{H}$ that need to be hit

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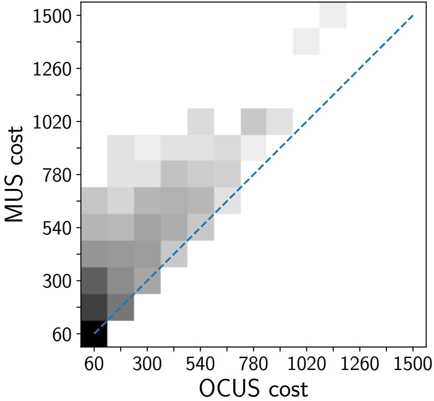
  - → Keep track of set of Satisfiable Subsets **SSs**
  - → Bootstrap $\mathcal{H} \leftarrow \{\mathcal{F} \setminus \mathcal{S} | \mathcal{S} \in \textbf{SSs}\}$

In practice!

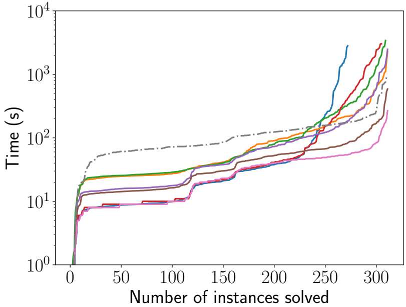
- Incremental OCUS works with the full unsatisfiable formula of step 0
  - $S \wedge E_{end} \wedge \{\neg n | n \in E_{end} \setminus E_0\}$
- Initialize hitting set solver once and modify objective at every explanation step i such that:
  - Underived literal cannot be taken
  - Negated literals already explained cannot selected

Results - Explanation quality

MUS
↓
OCUS



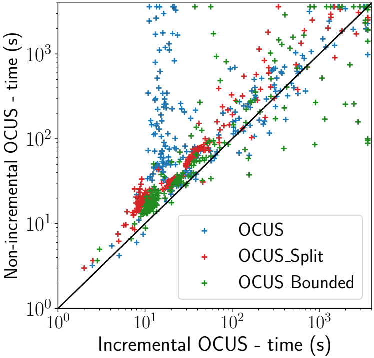
Results - Correction Subset Enumeration



Algorithm: $\textsc{ocus}(\mathcal{F}, f, p)$

$\mathcal{H} \leftarrow \emptyset$
while true **do**
  $\mathcal{S} \leftarrow \textsc{CondOptHittingSet}(\mathcal{H}, f, p)$
  if $\neg\mathrm{SAT}(\mathcal{S})$ **then**
    return $\mathcal{S}$
  end
  $\mathcal{H} \leftarrow$
  $\mathcal{H} \cup \textsc{Correction-Subsets}(\mathcal{S}, \mathcal{F})$
end

Results - Incrementality



Algorithm: $\text{OCUS}(\mathcal{F}, f, p)$

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  $\mathcal{H} \leftarrow \mathcal{H} \cup \text{Correction-Subsets}(\mathcal{S}, \mathcal{F})$

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Conclusion

We introduce Optimal Constrained Unsatisfiable Subsets (OCUS) problem and solved it using the implicit hitting set duality between MCSes and MUSes.

Optimality Cost-function quantifies explanation difficulty.

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Optimality Cost-function quantifies explanation difficulty.

Constrainedness Impose structural constraints on the form of the explanations.

Incrementality Reuse-information between successive explanation calls.

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- Explaining scheduling, configuration problems .... and puzzles 😎
- Debugging *unsatisfiable models* with preferences on the constraints

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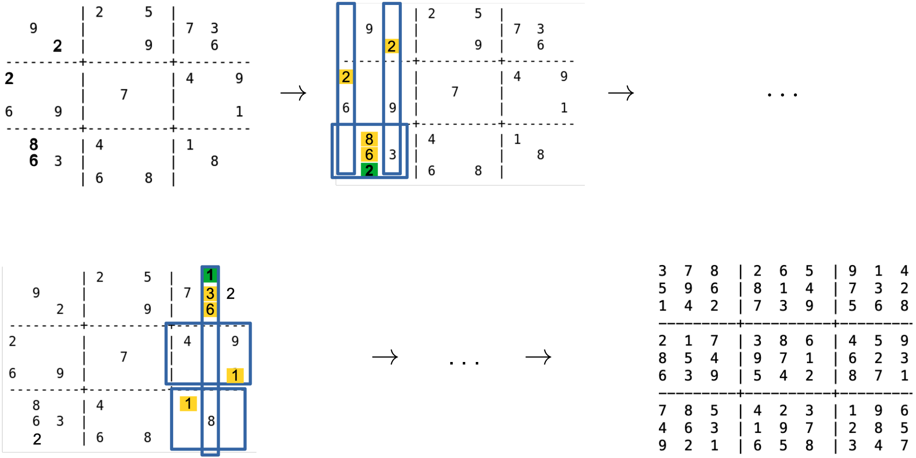
- Debugging *unsatisfiable models* with preferences on the constraints
- Stepwise explaining unsatisfiability
- Explaining Optimality for Constraint Optimization Problems
  - Why is the objective value not better ?

Future work

- What is a good cost-function to quantify how difficult an explanation is ? (from humans)
- Explaining optimization (different types of “why” queries); close relation to Explainable AI Planning Fox *et al.* [2017]
- Scaling up (approximate algorithms; decomposition of explanation search)

Future work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5 9 6 | 8 1 4 | 7 3 2
1 4 2 | 7 3 9 | 5 6 8
2 1 7 | 3 8 6 | 4 5 9
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6 3 9 | 5 4 2 | 8 7 1
7 8 5 | 4 2 3 | 1 9 6
4 6 3 | 1 9 7 | 2 8 5
9 2 1 | 6 5 8 | 3 4 7

This is joint work with ...

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ARTIFICIAL INTELLIGENCE RESEARCH GROUP

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