IMPLEMENTING CUMULATIVE FUNCTIONS FOR CONDITIONAL TASK INTERVALS USING A GENERALIZED CUMULATIVE CONSTRAINT

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2: University of Maroua, Faculty of Sciences, Department of Mathematics and Computer Science

Artificial Intelligence & Algorithms

icteam

UCLouvain

SCHEDULING WITH RESOURCES

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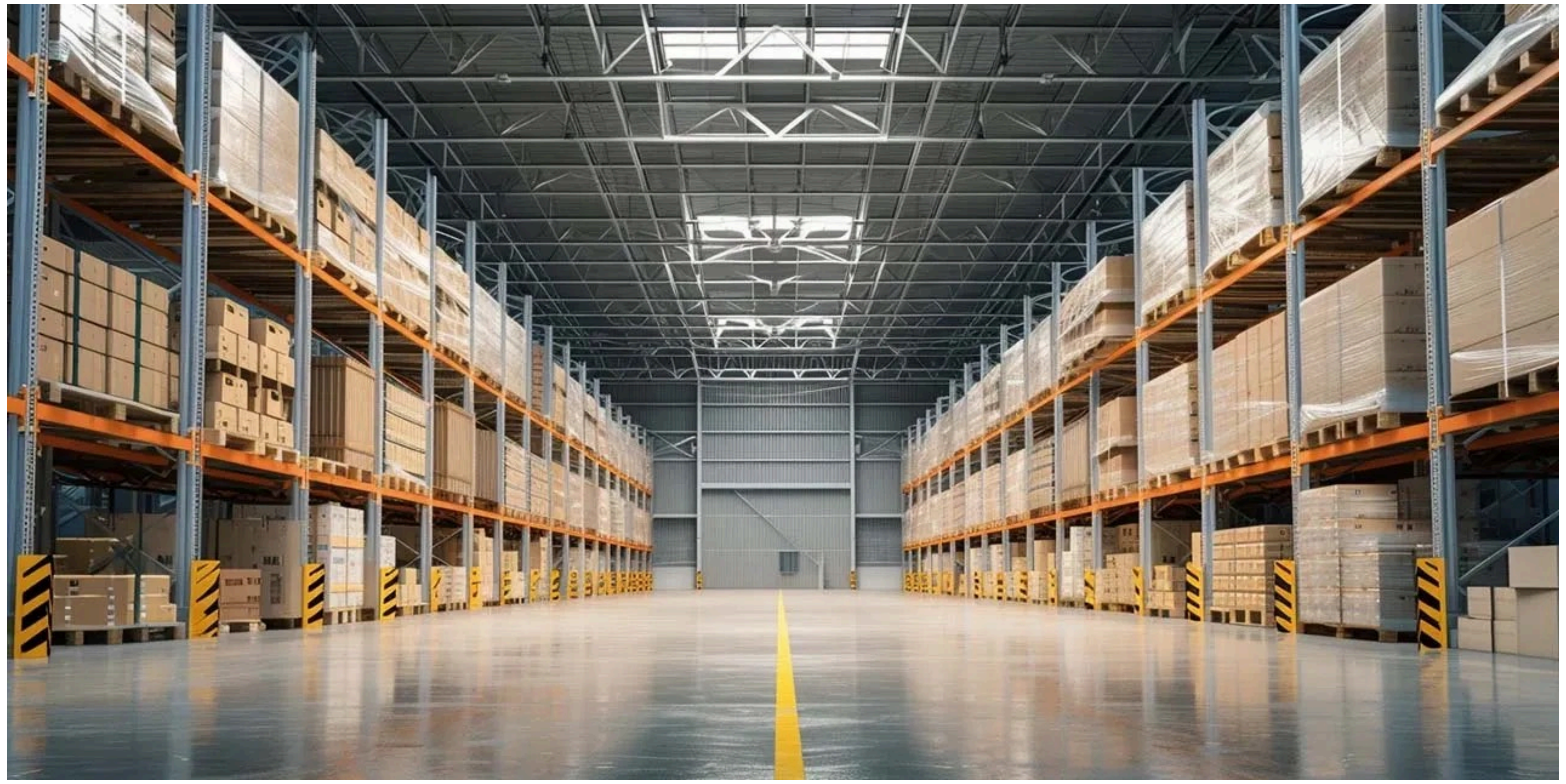

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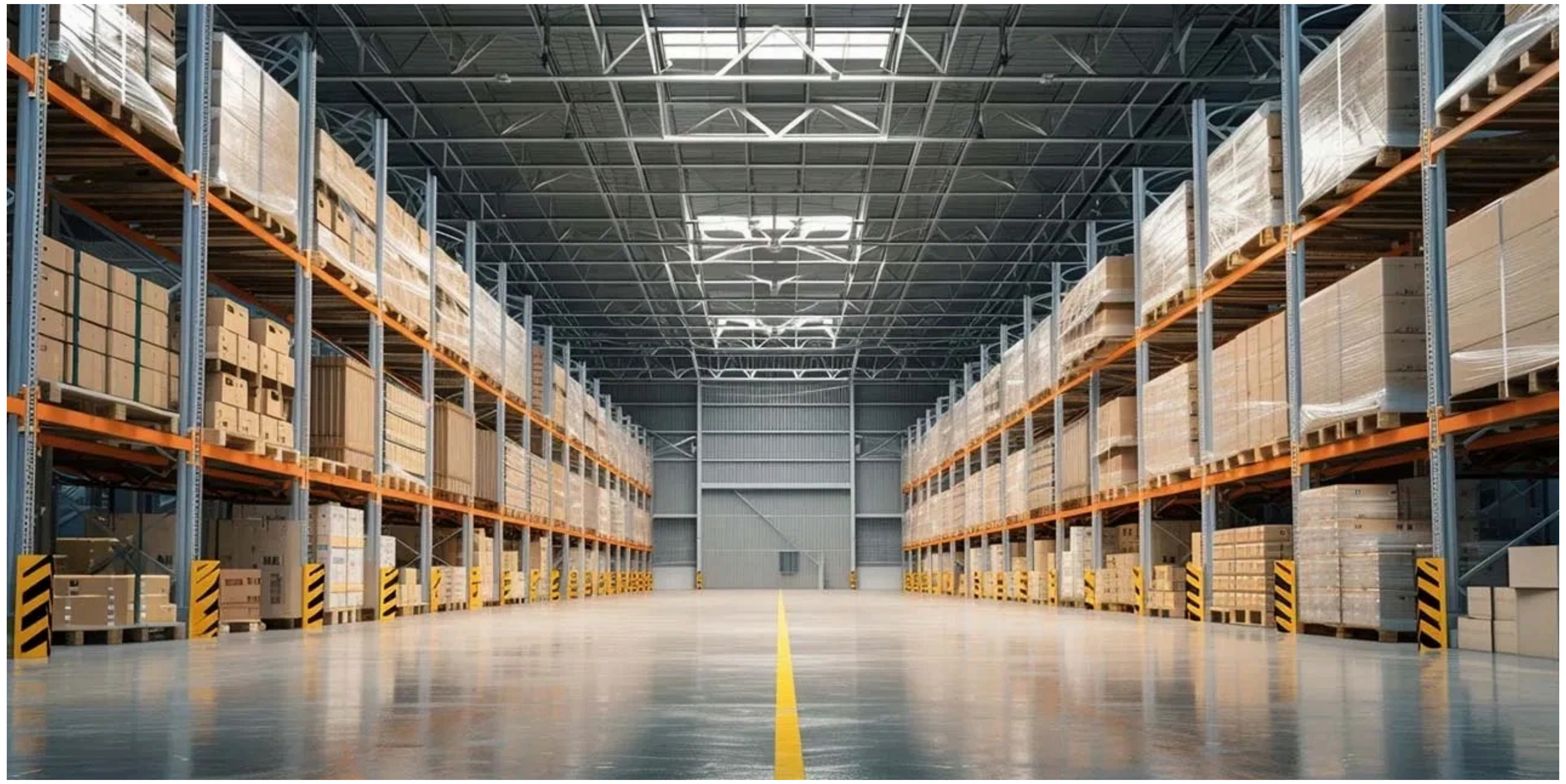

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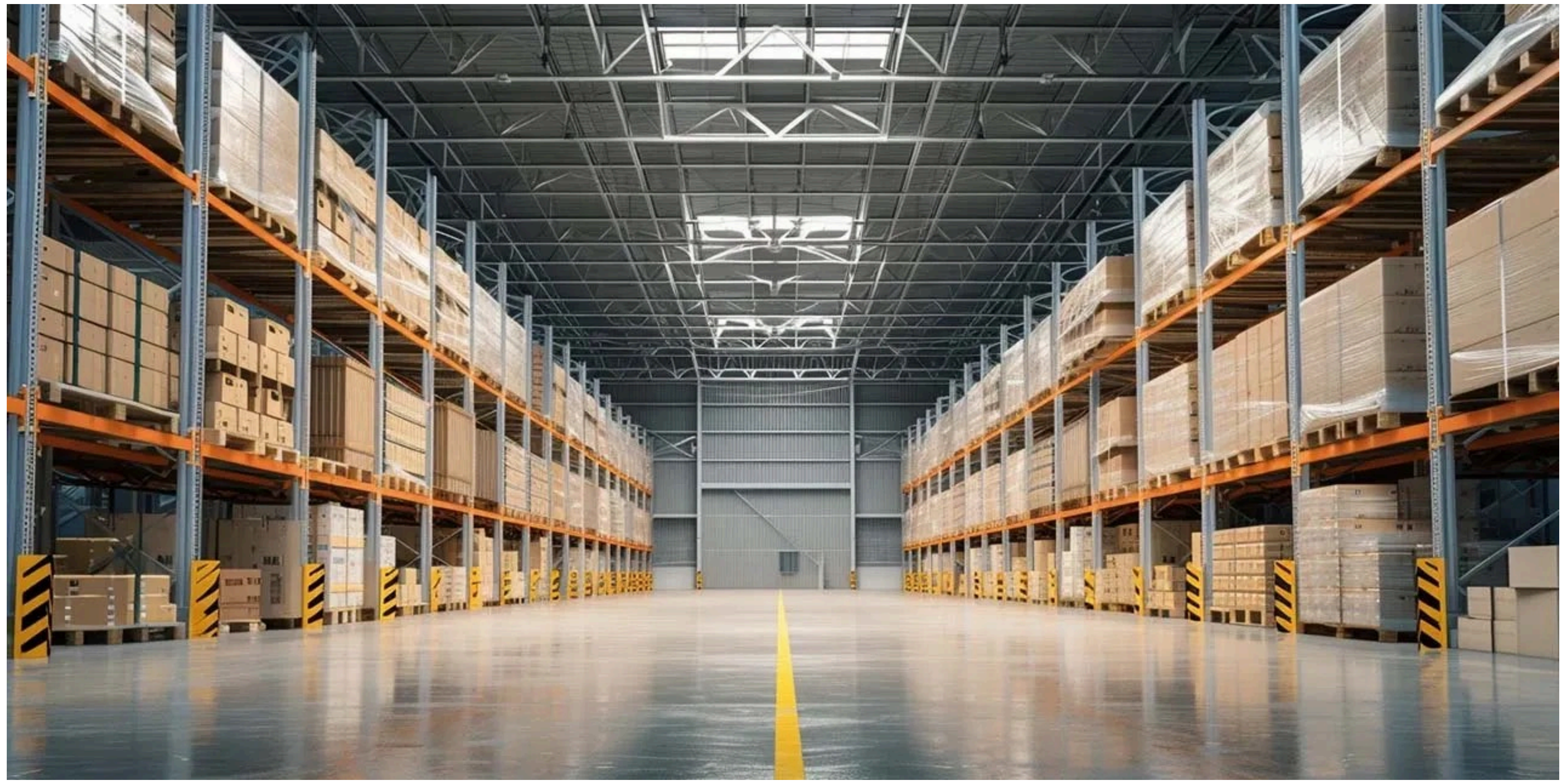

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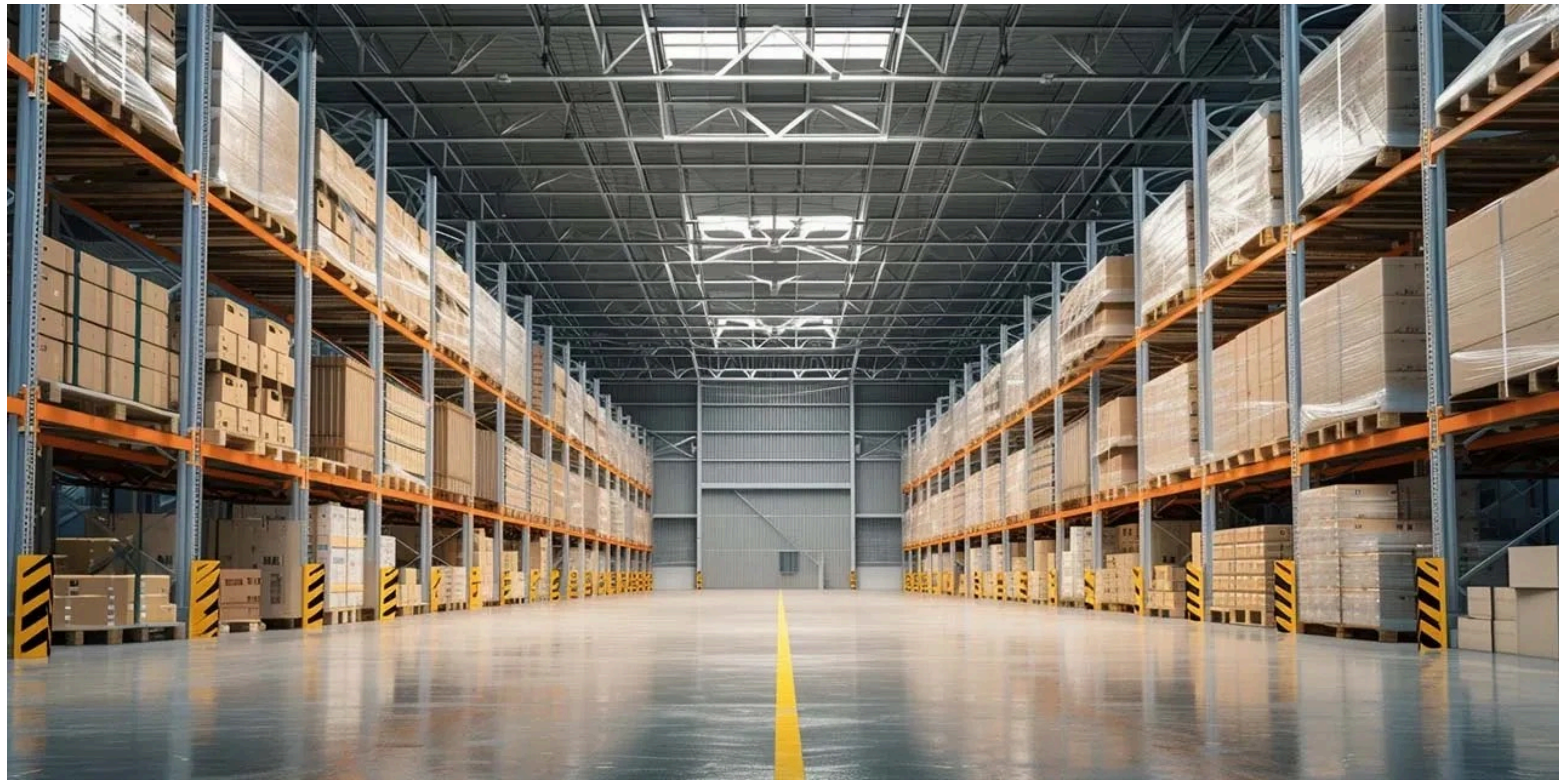

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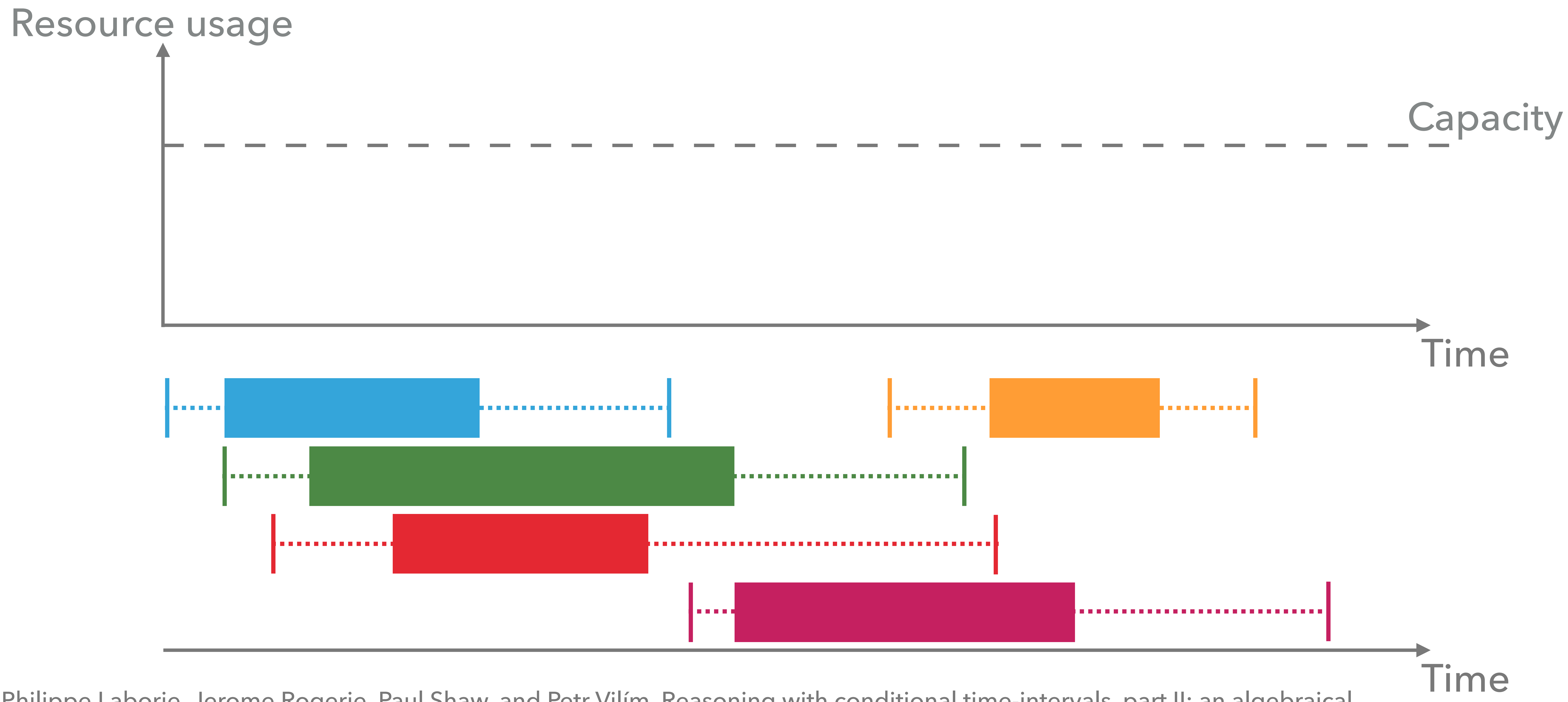

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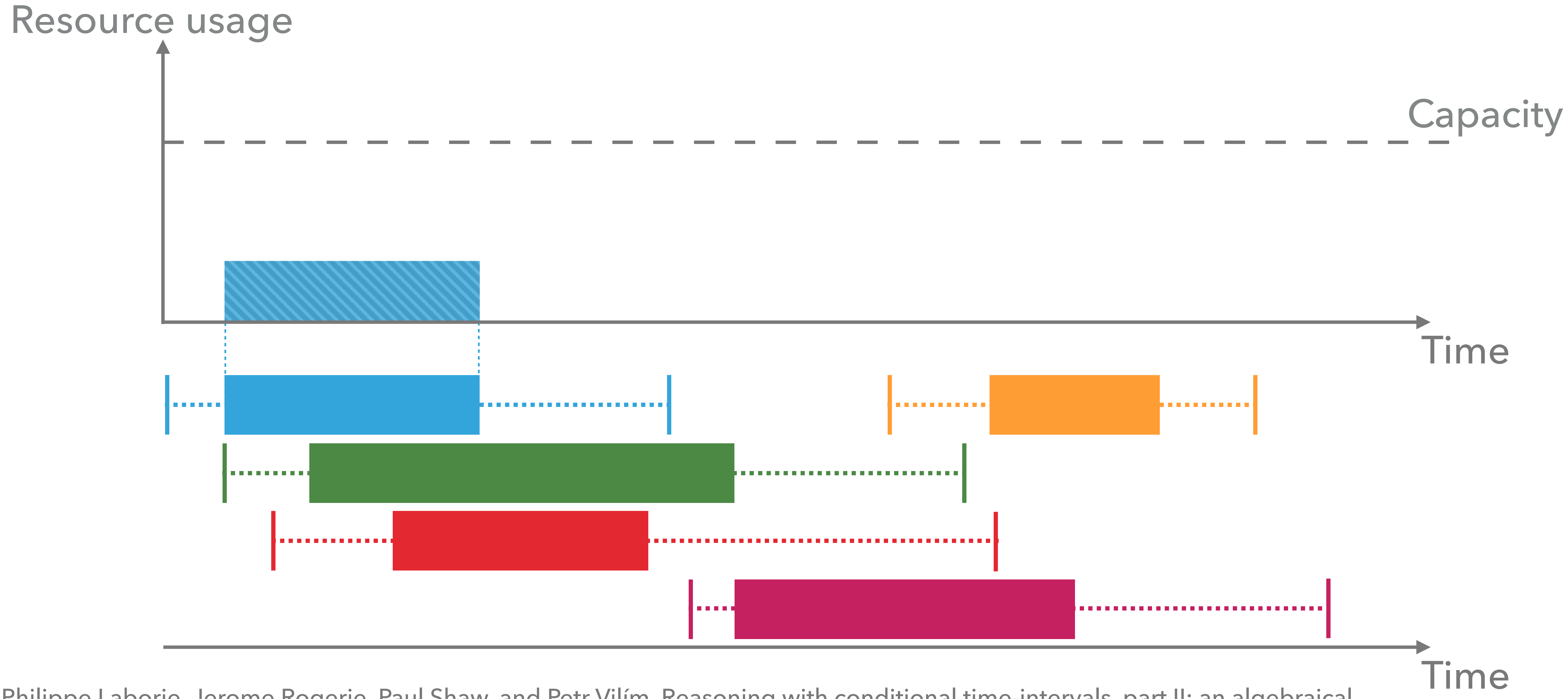

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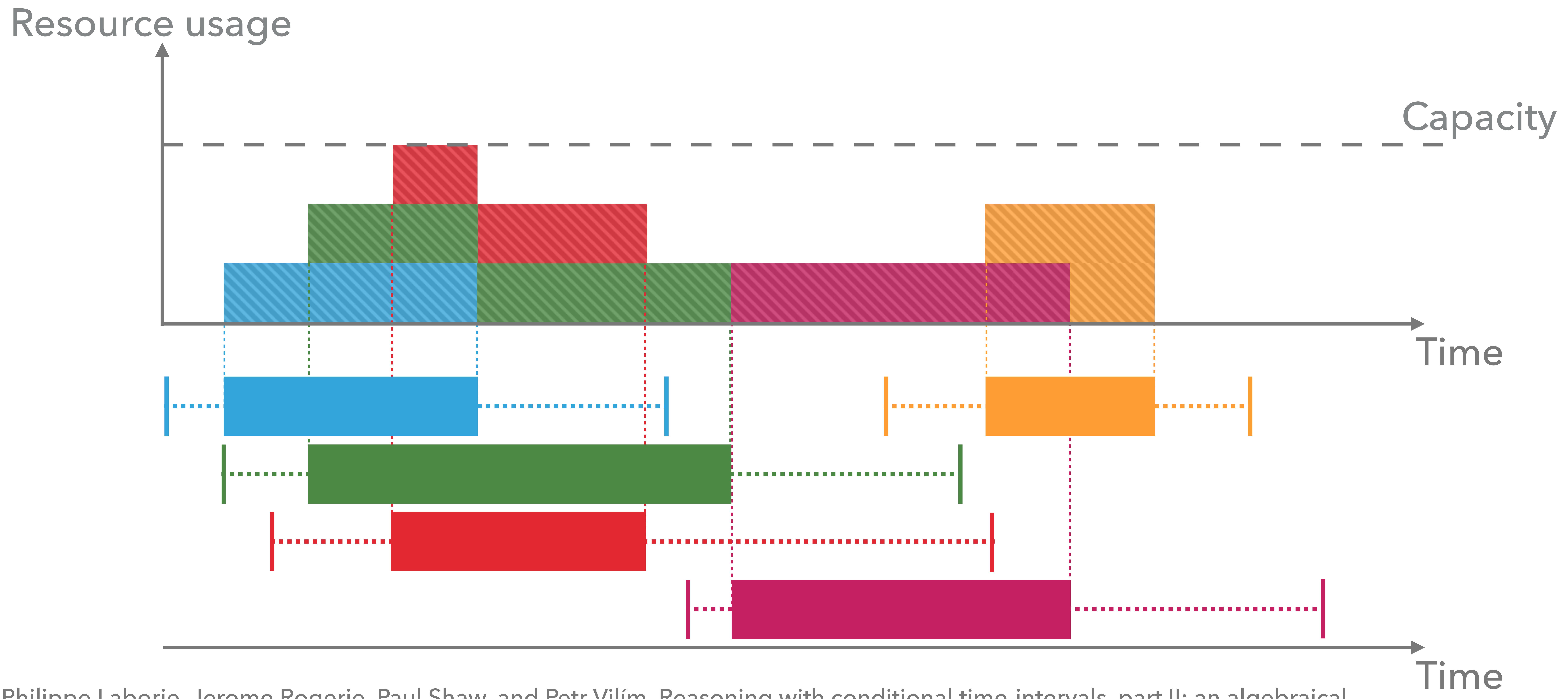

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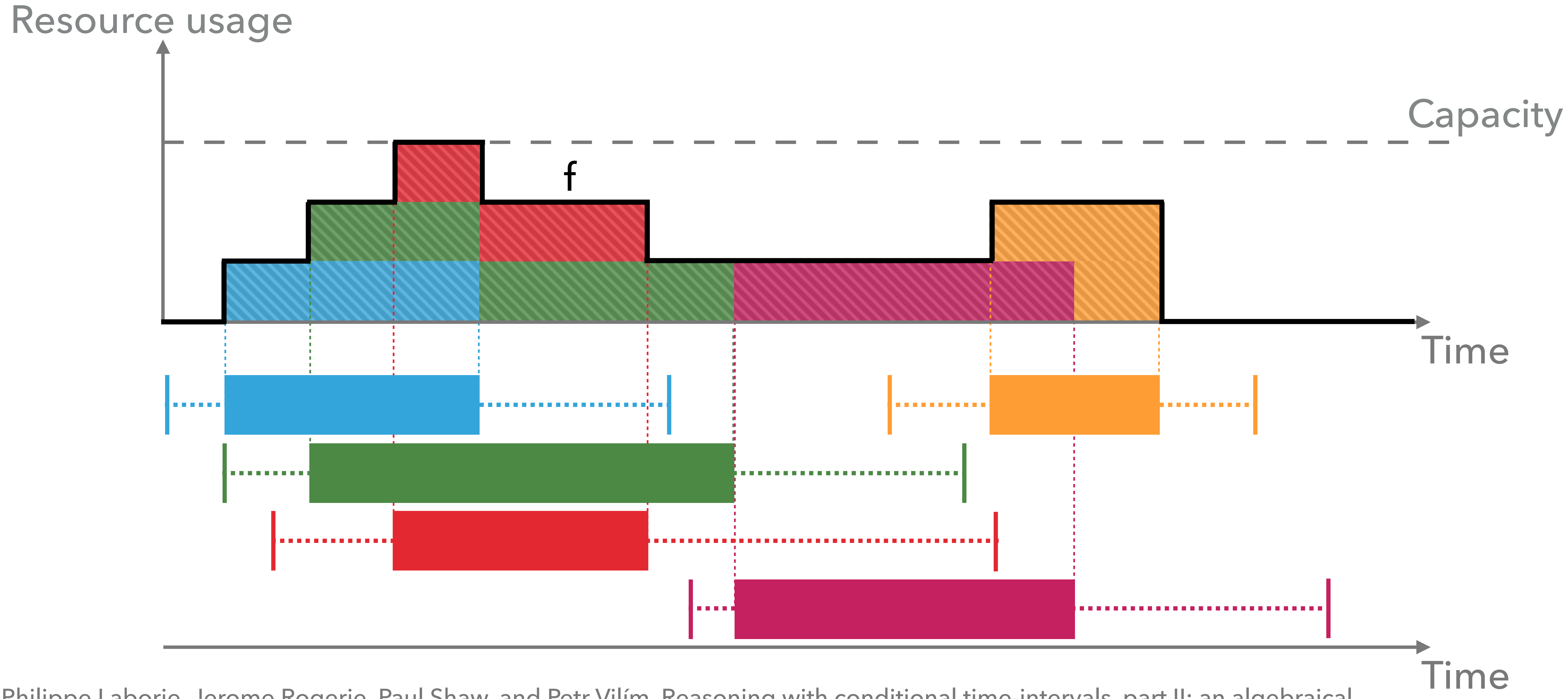

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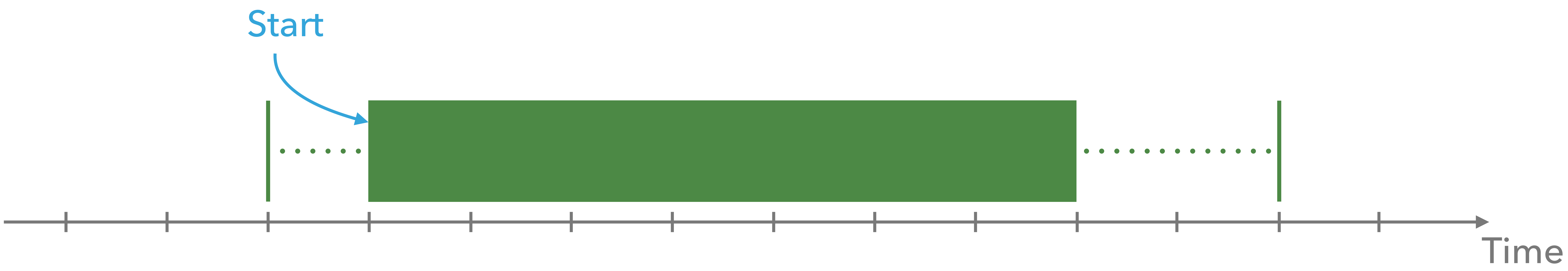

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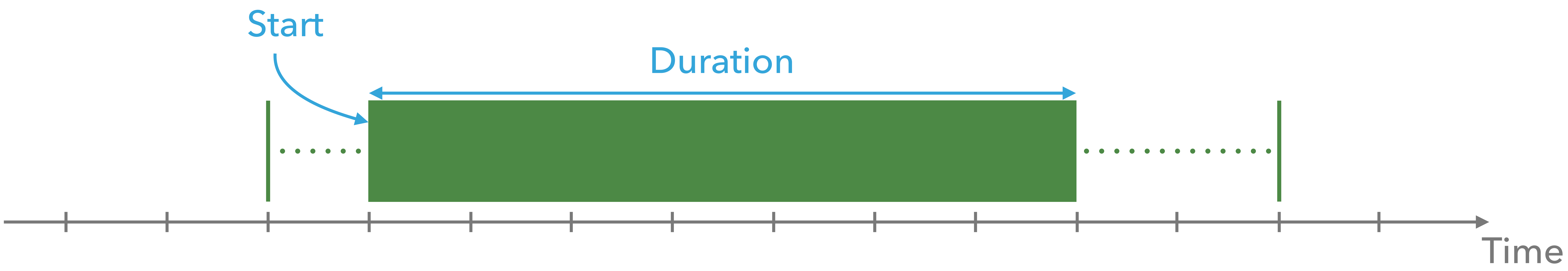

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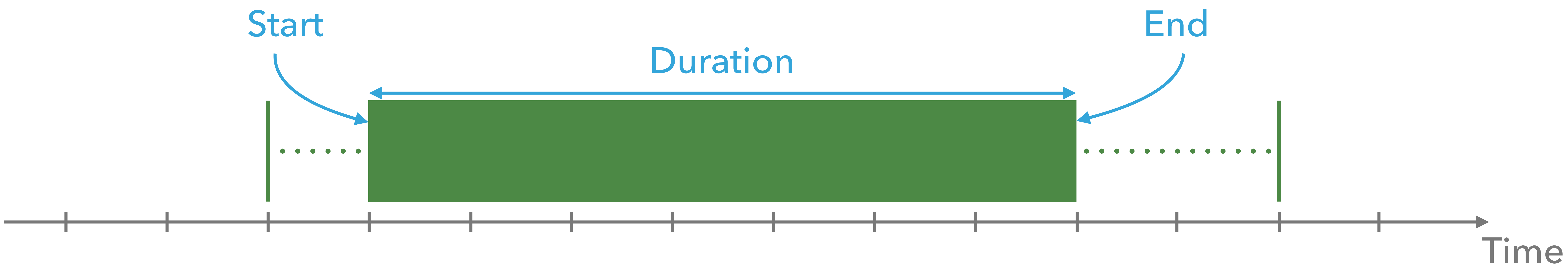

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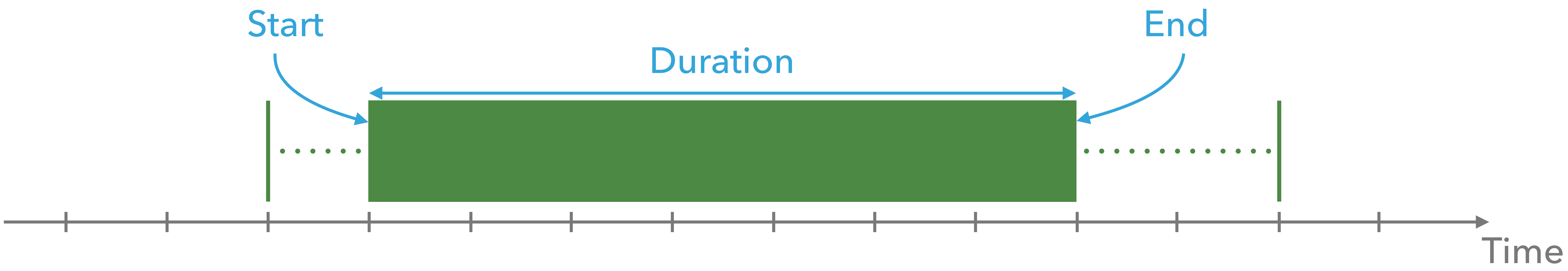

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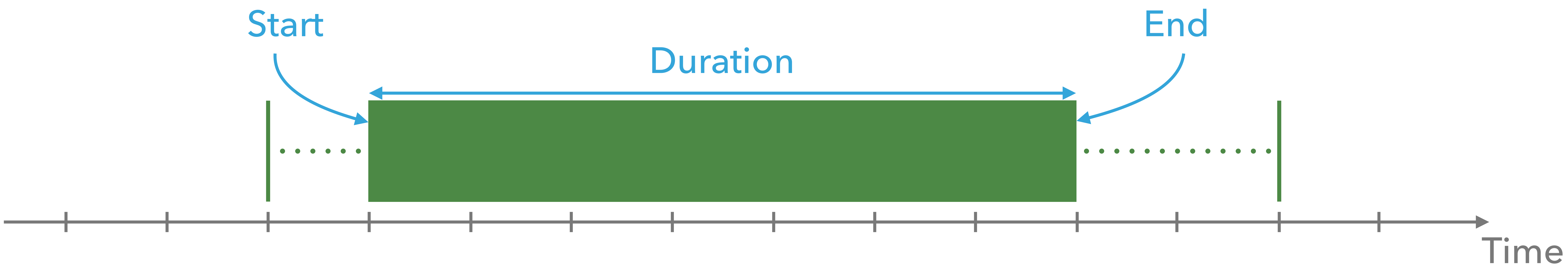

- **Optional**: can take the value ⊥ to indicate that the interval is not present
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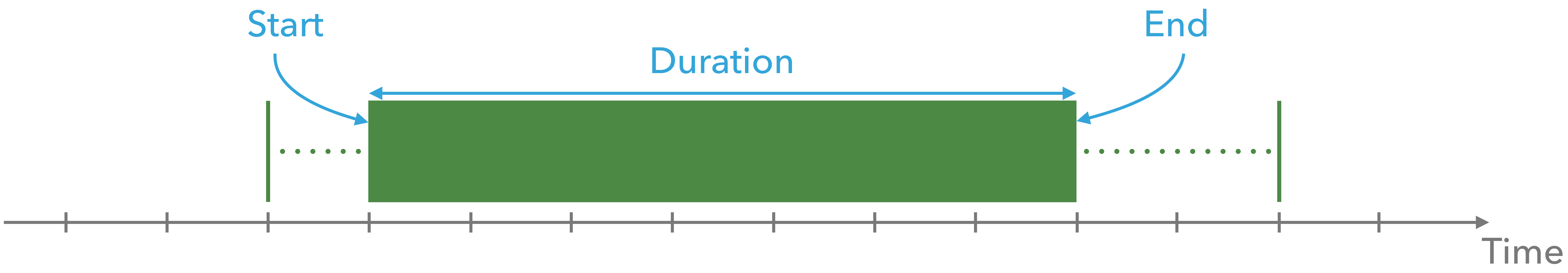

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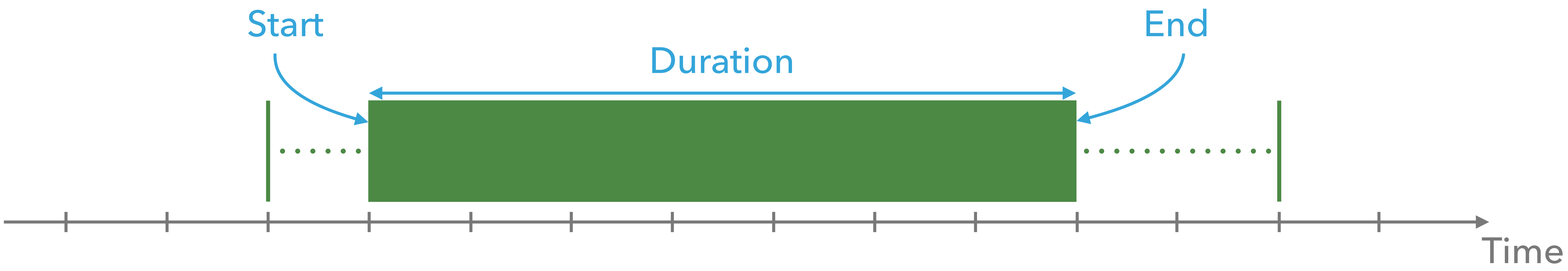

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  - **start = {}** ∨ **duration = {}** ∨ **end = {}** ⇒ ⊥

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ELEMENTARY CUMULATIVE FUNCTIONS

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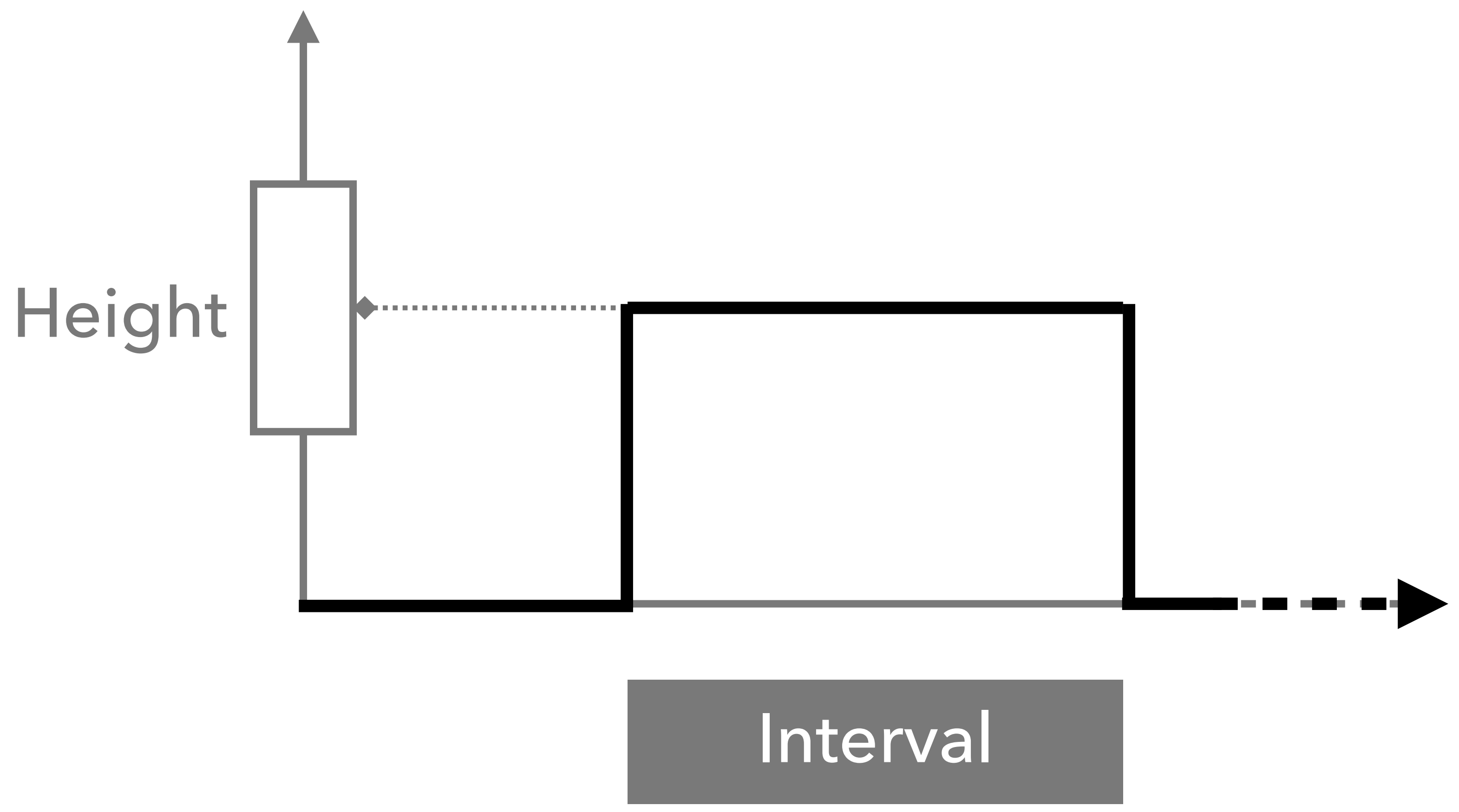

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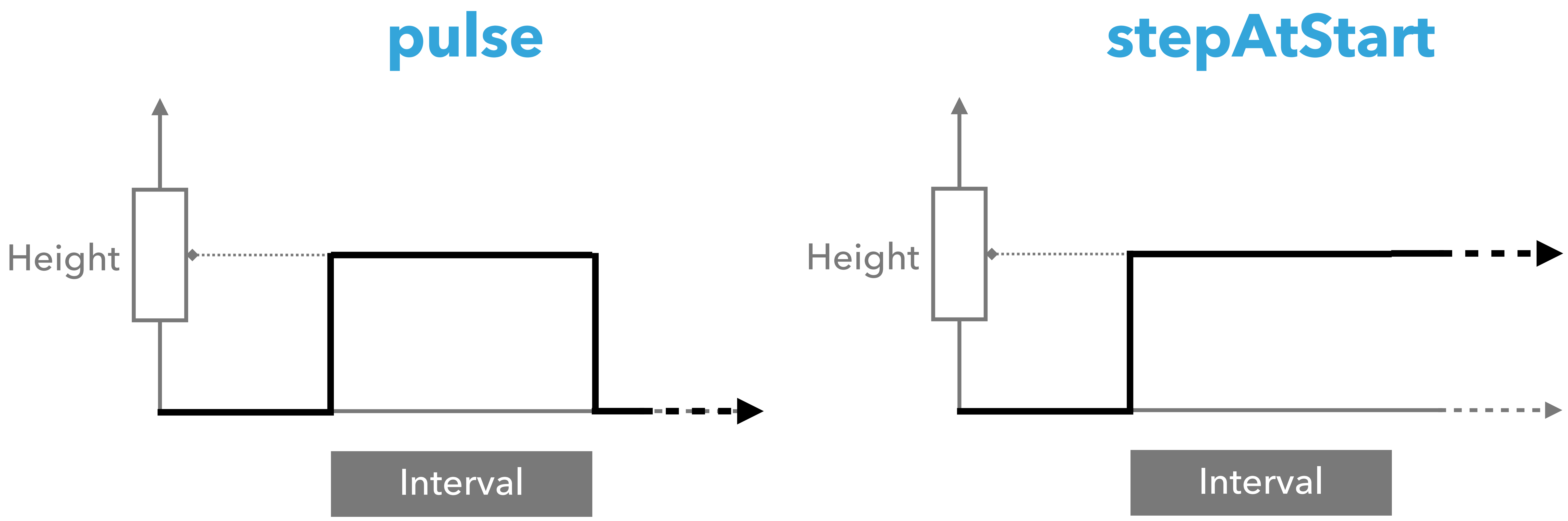

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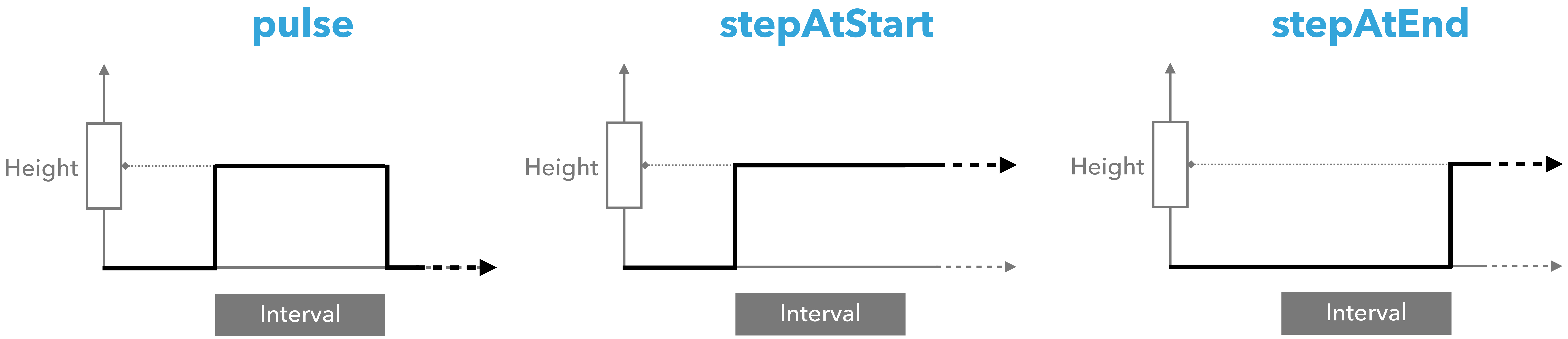

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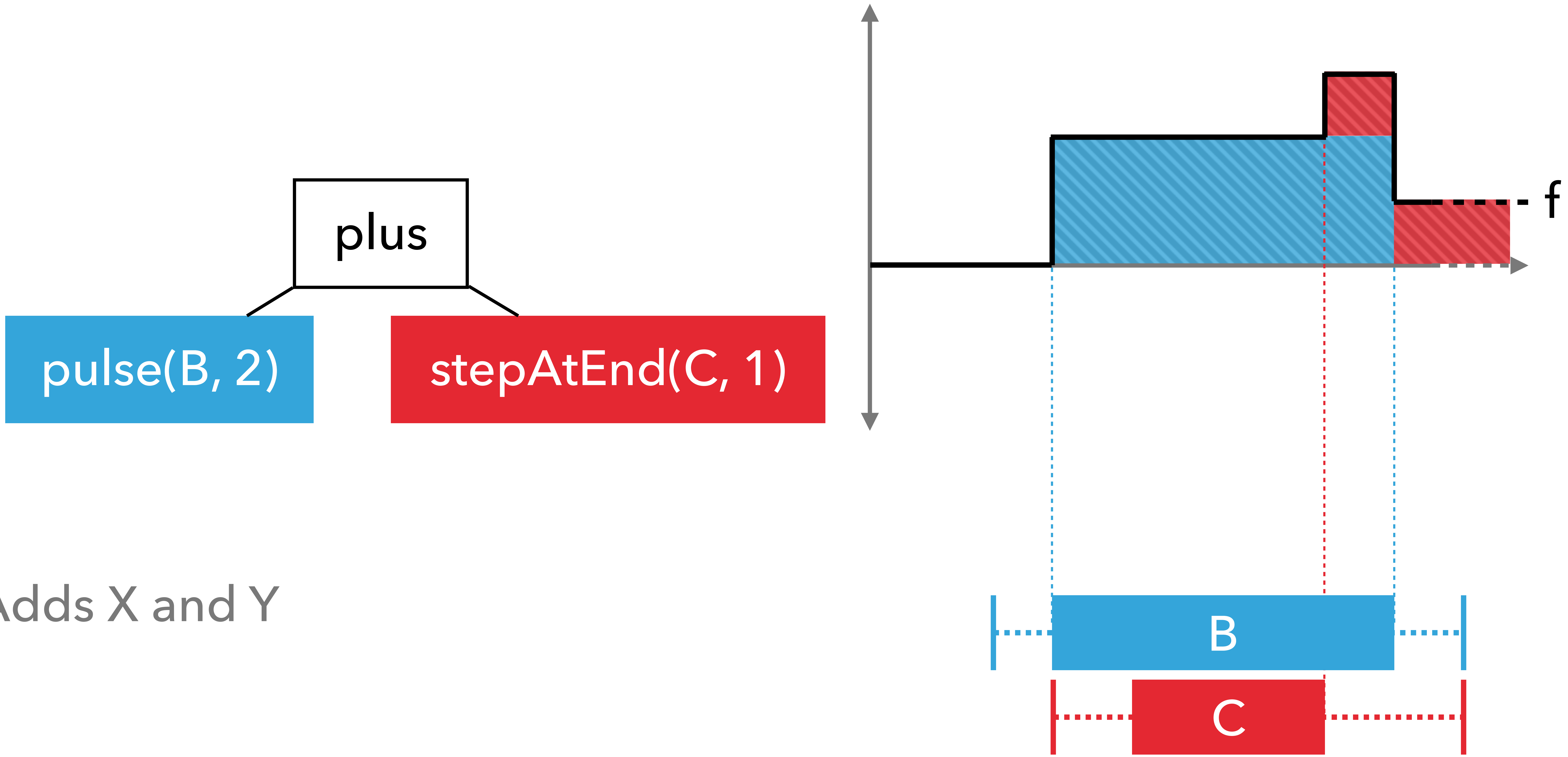

- **Plus(X, Y)** : Adds X and Y

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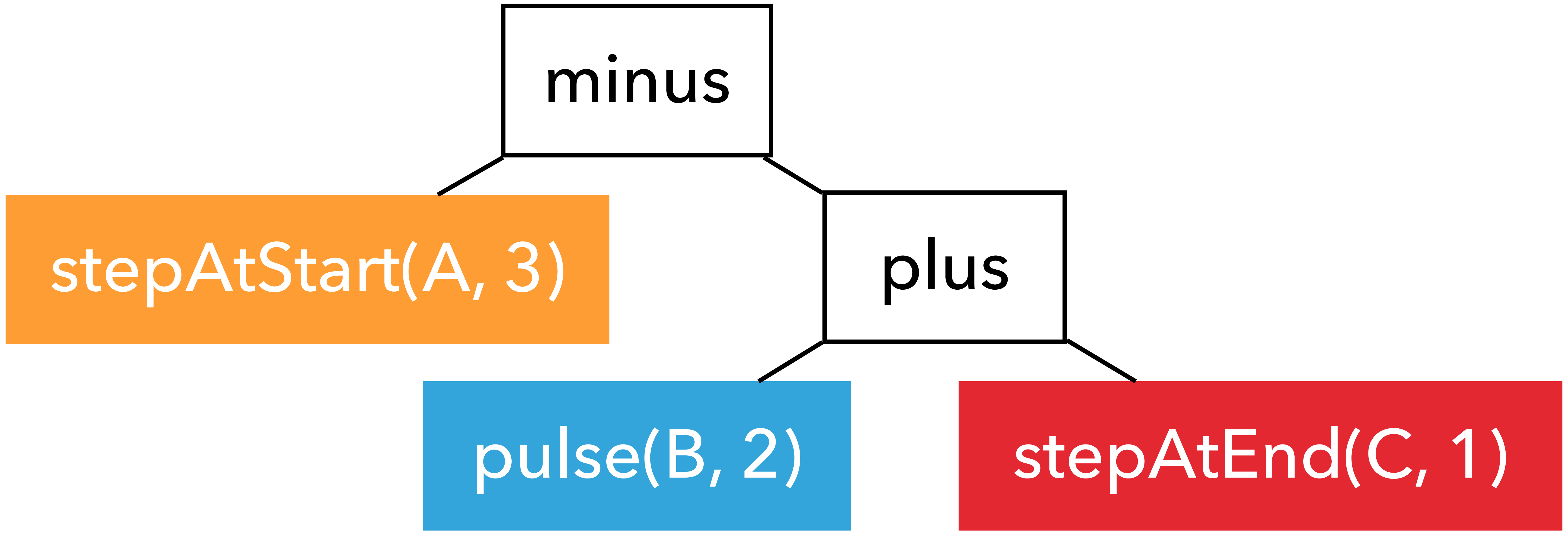

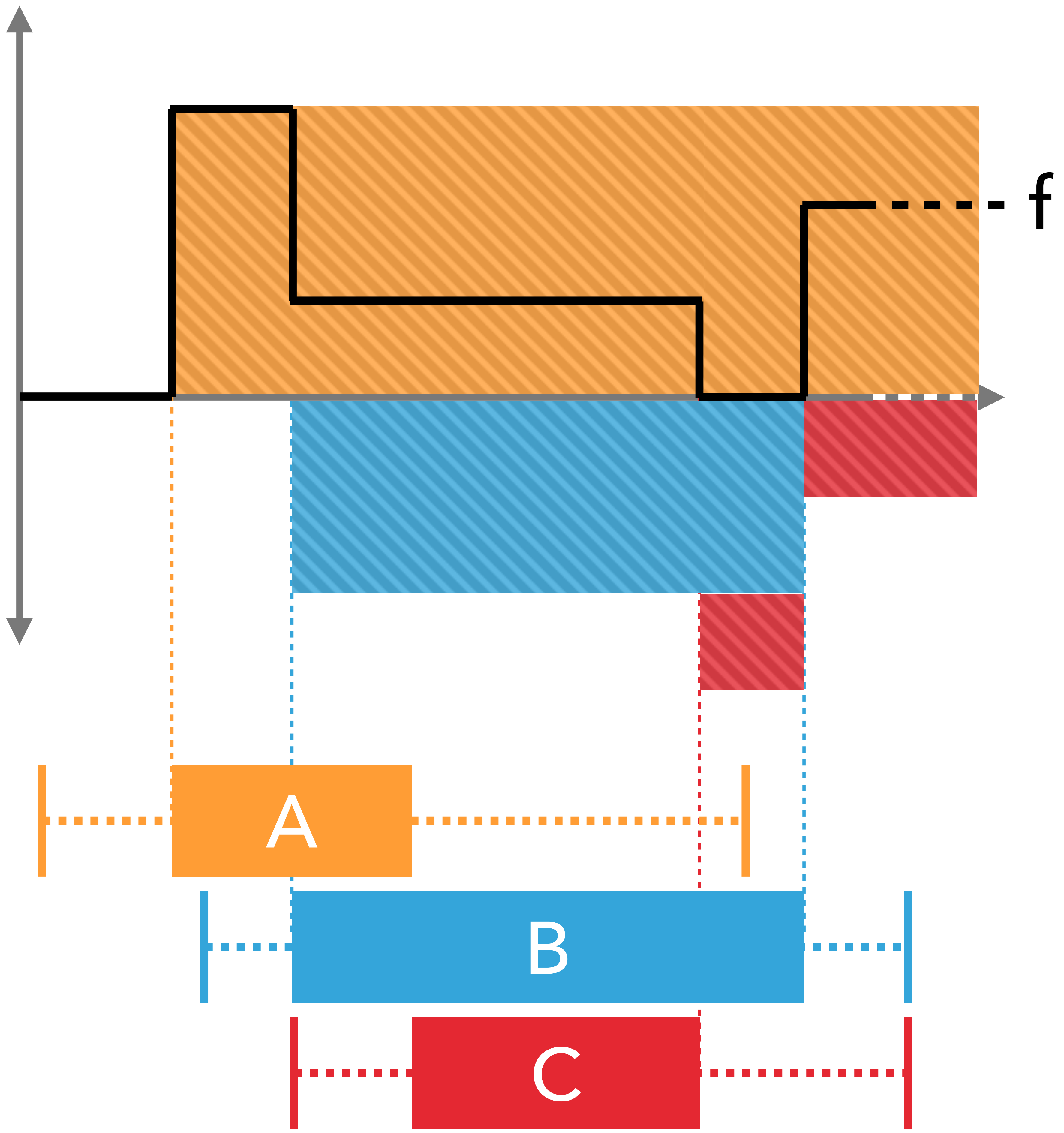

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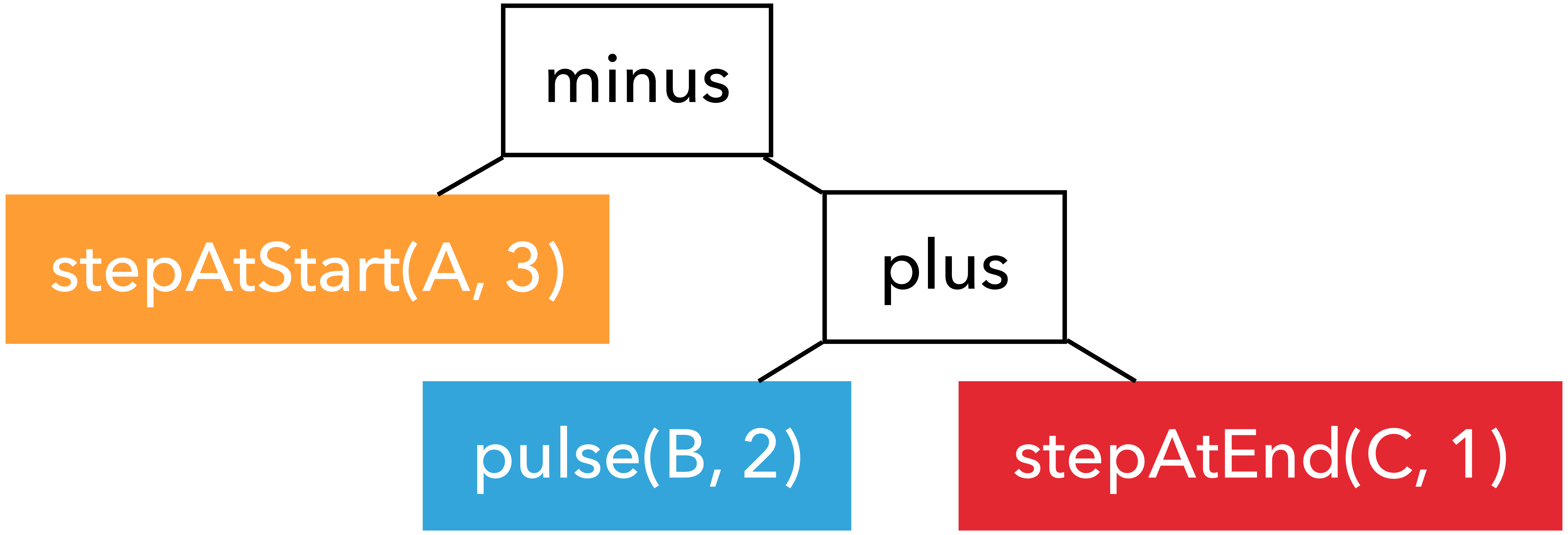

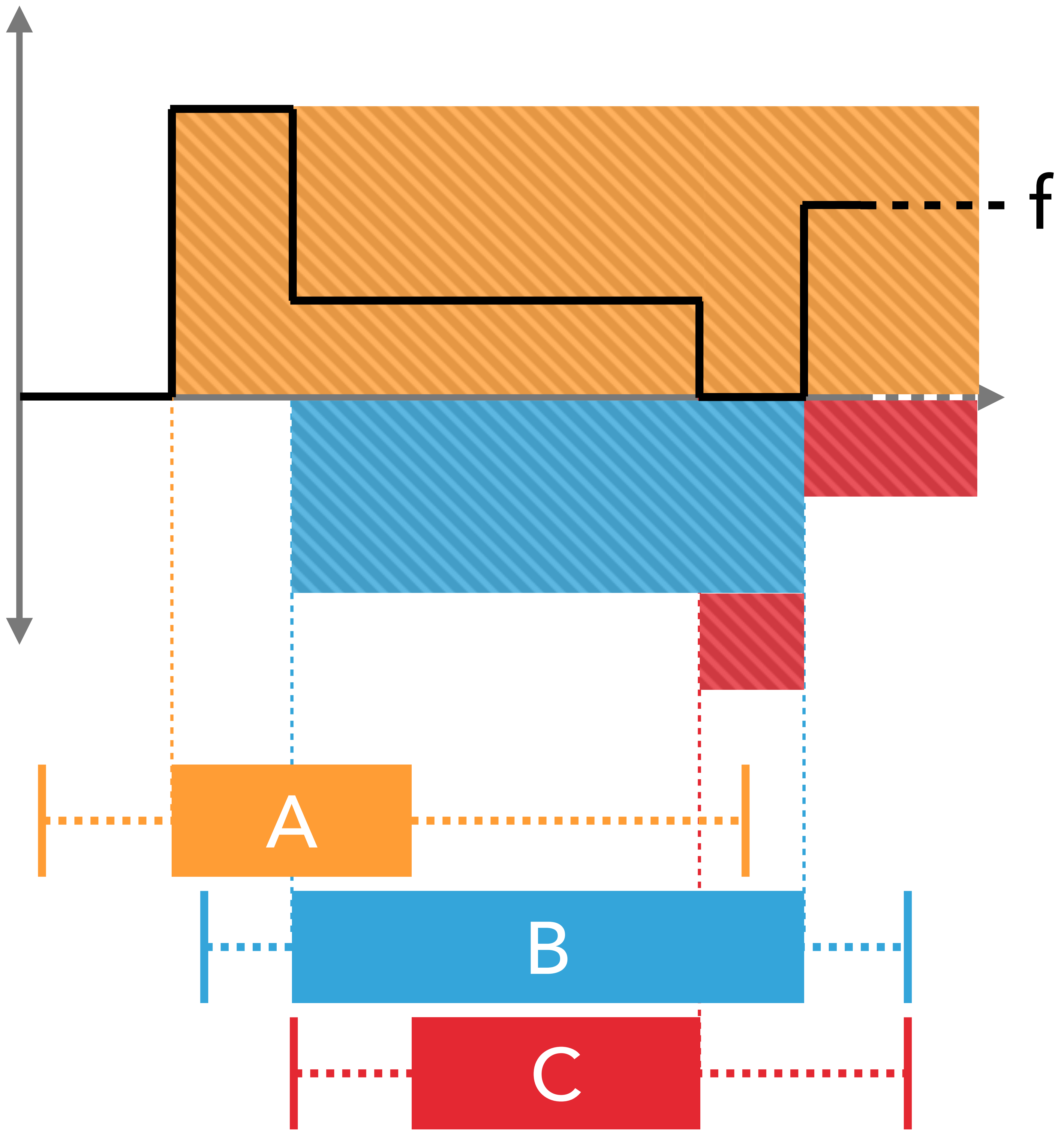

- **Plus(X, Y)** : Adds X and Y
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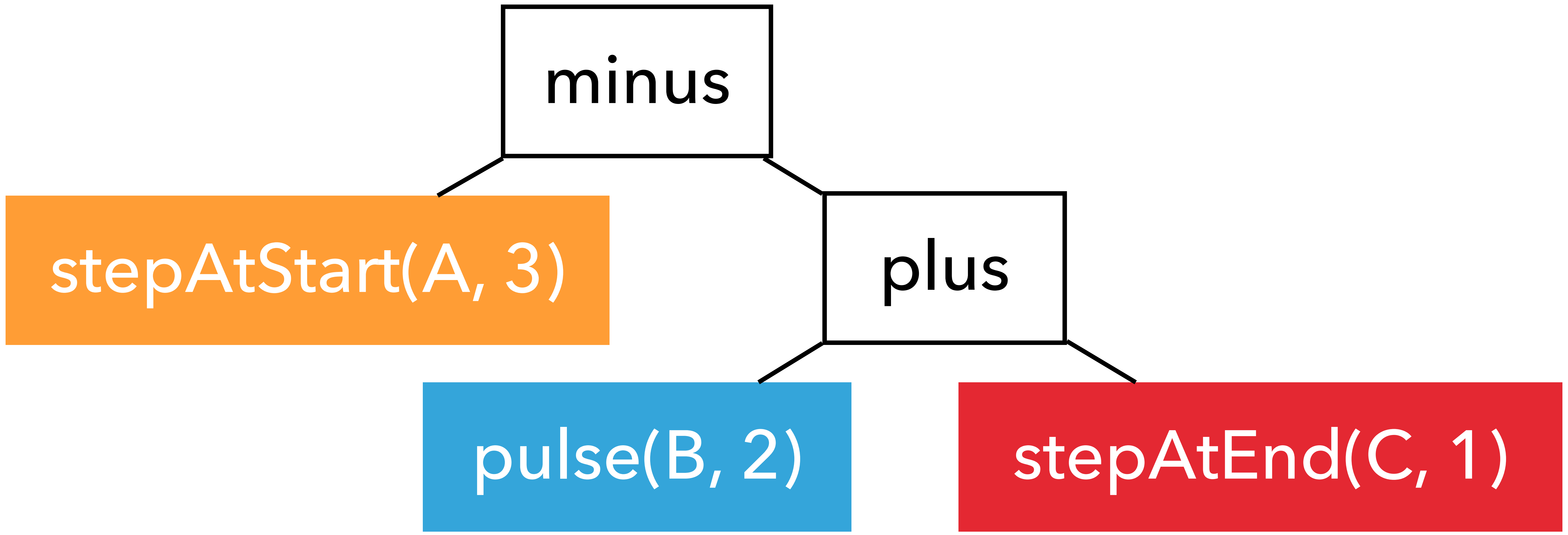
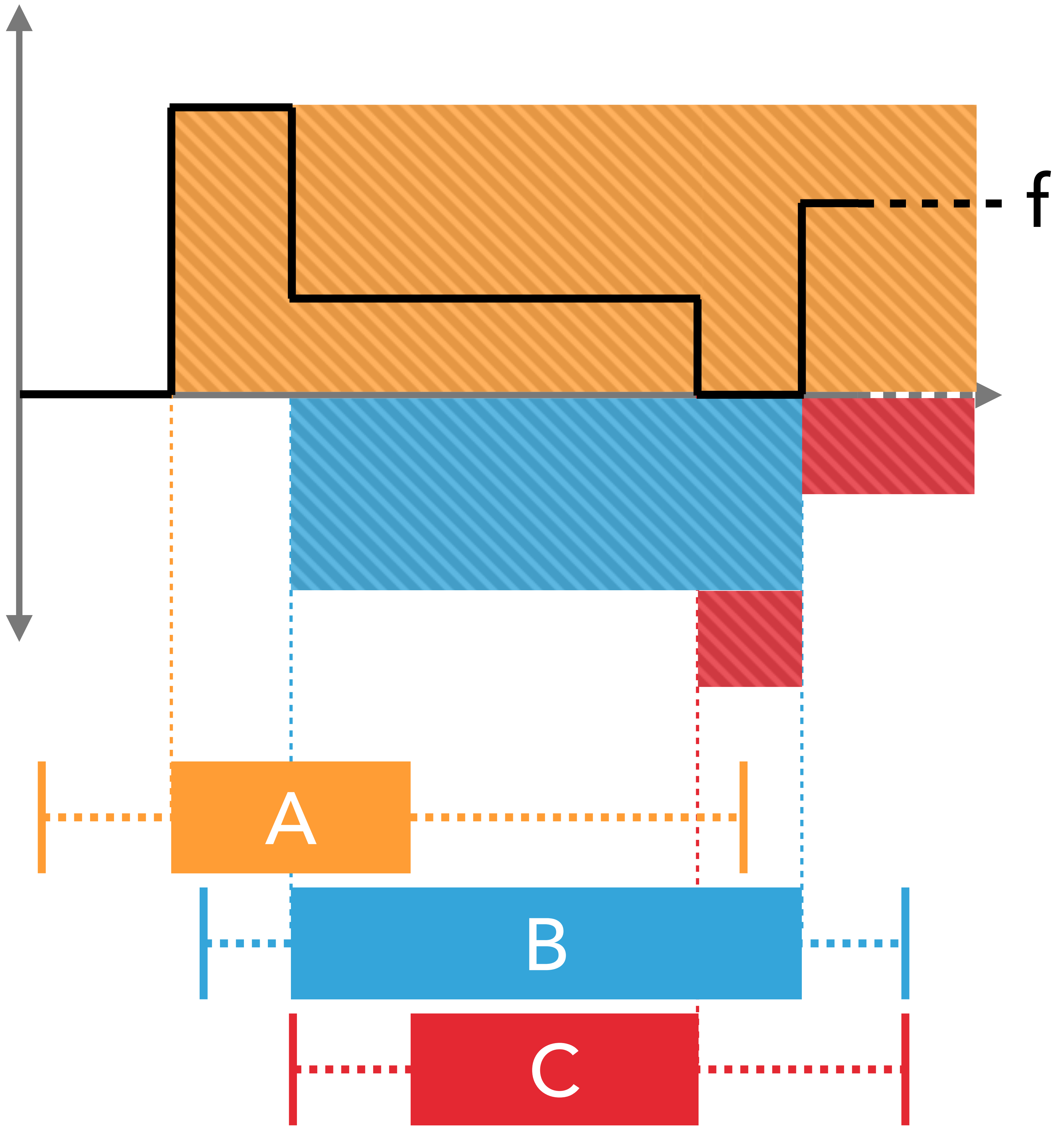
- Forms an **abstract syntax tree** (AST)

CONSTRAINTS ON CUMULATIVE FUNCTIONS





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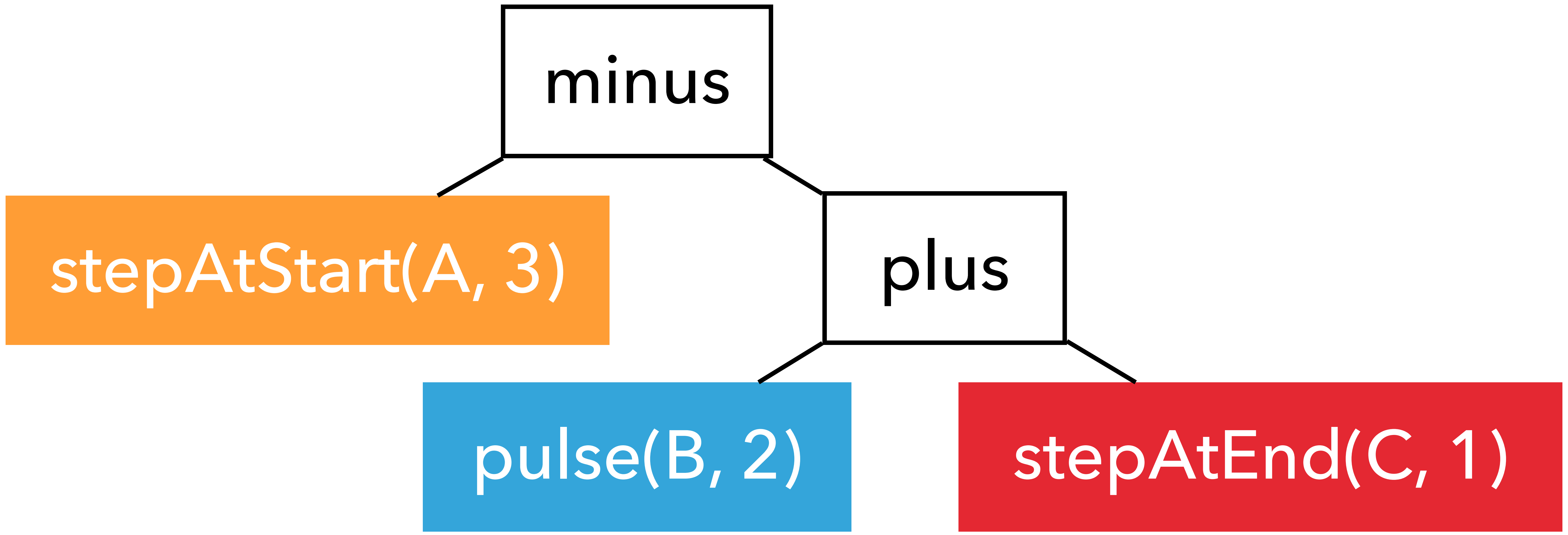

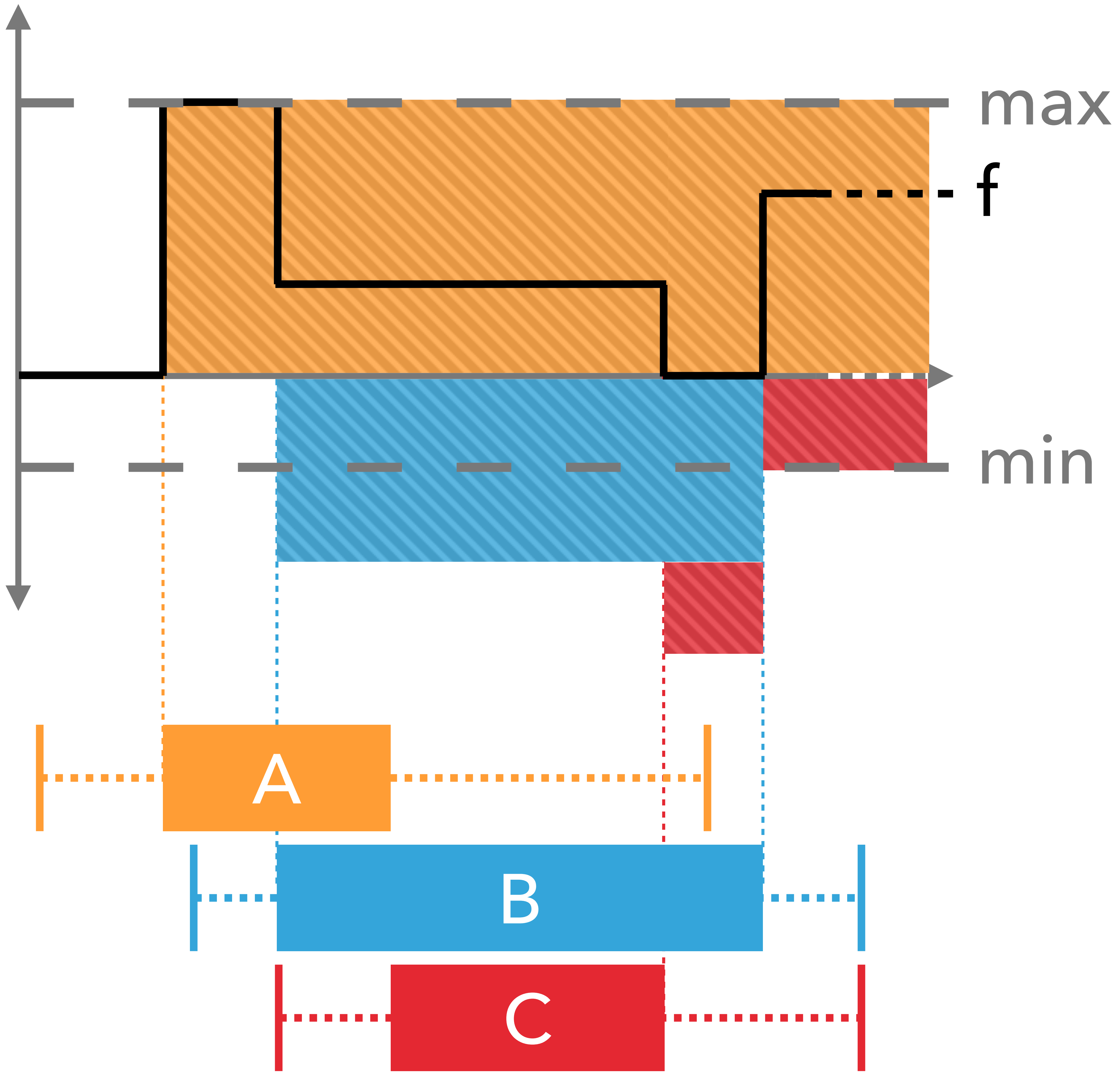

- **alwaysIn(f, min, max) : min ≤ f ≤ max**

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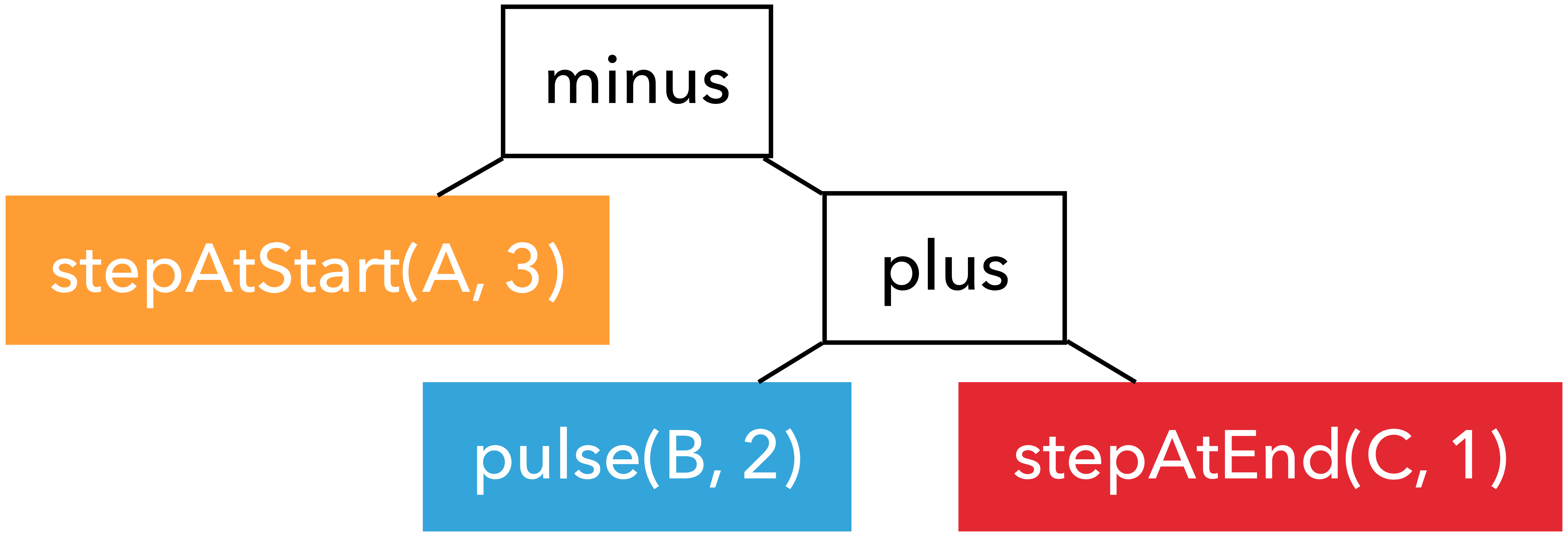

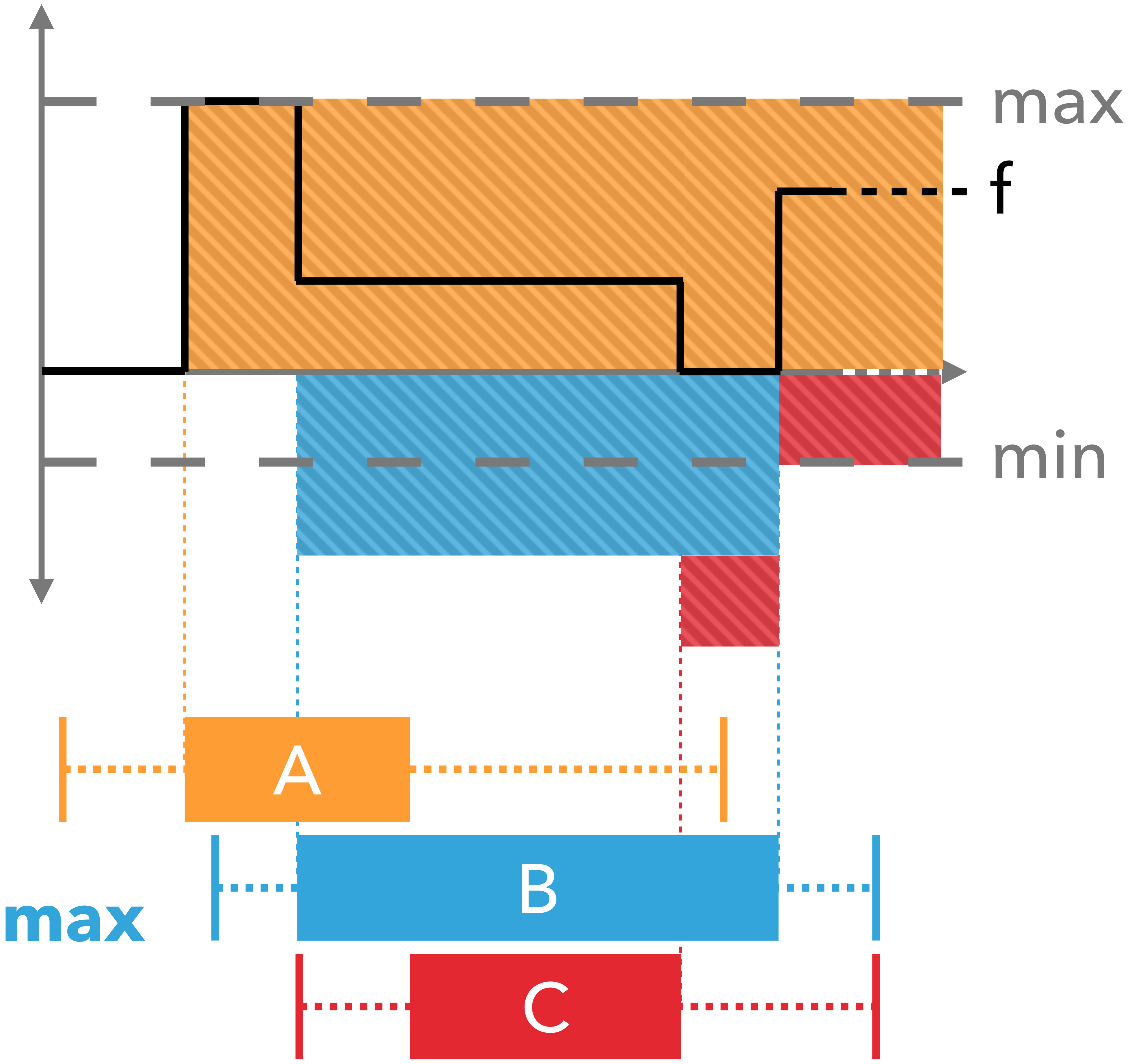

- **alwaysIn(f, min, max)** : **min ≤ f ≤ max**
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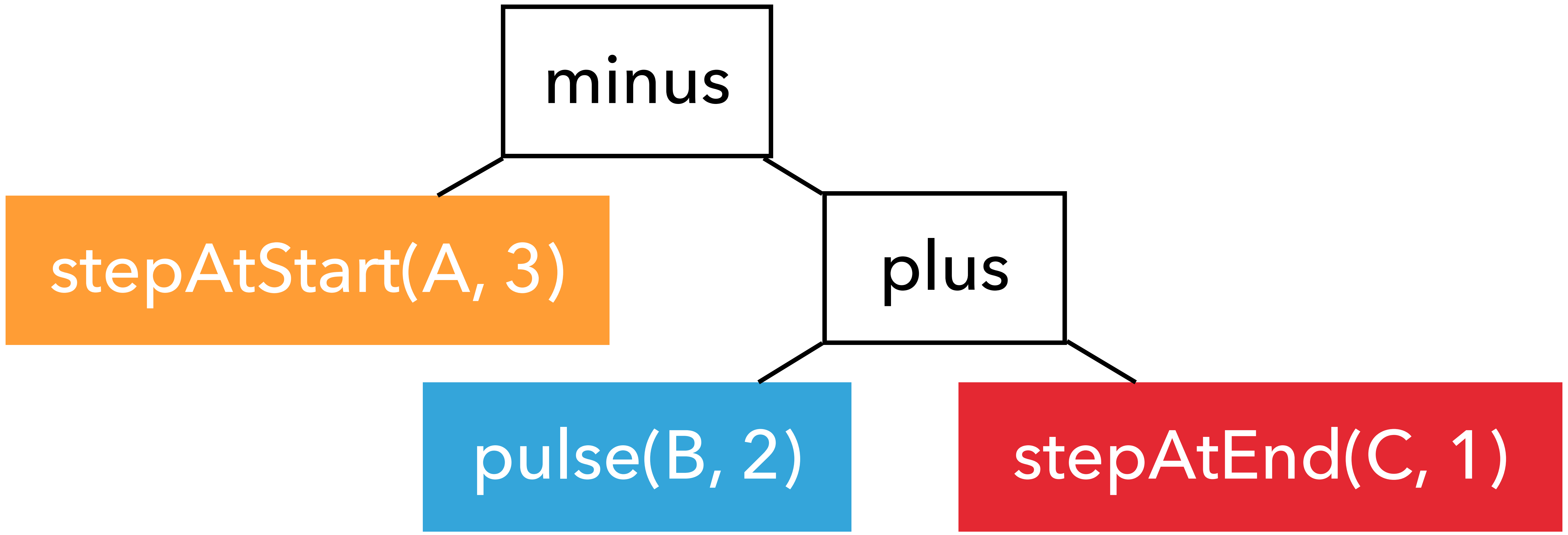

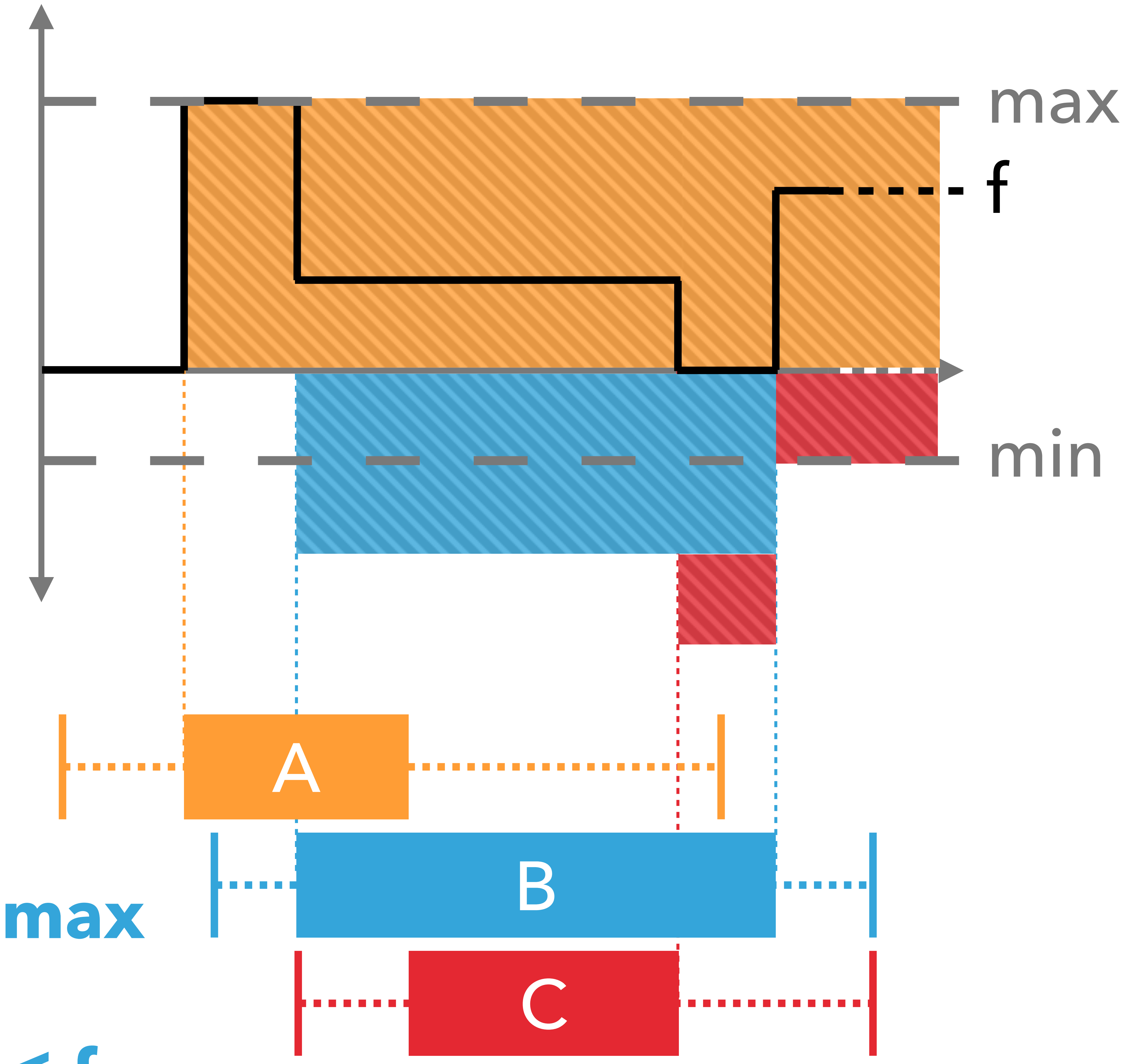

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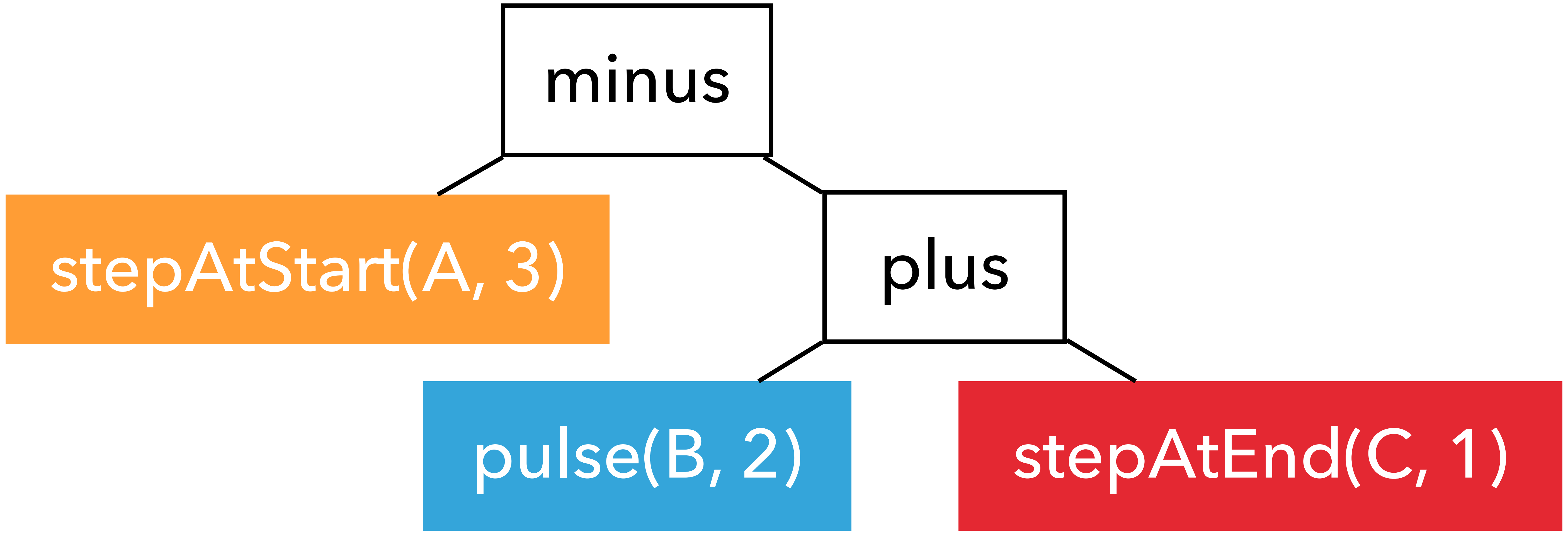

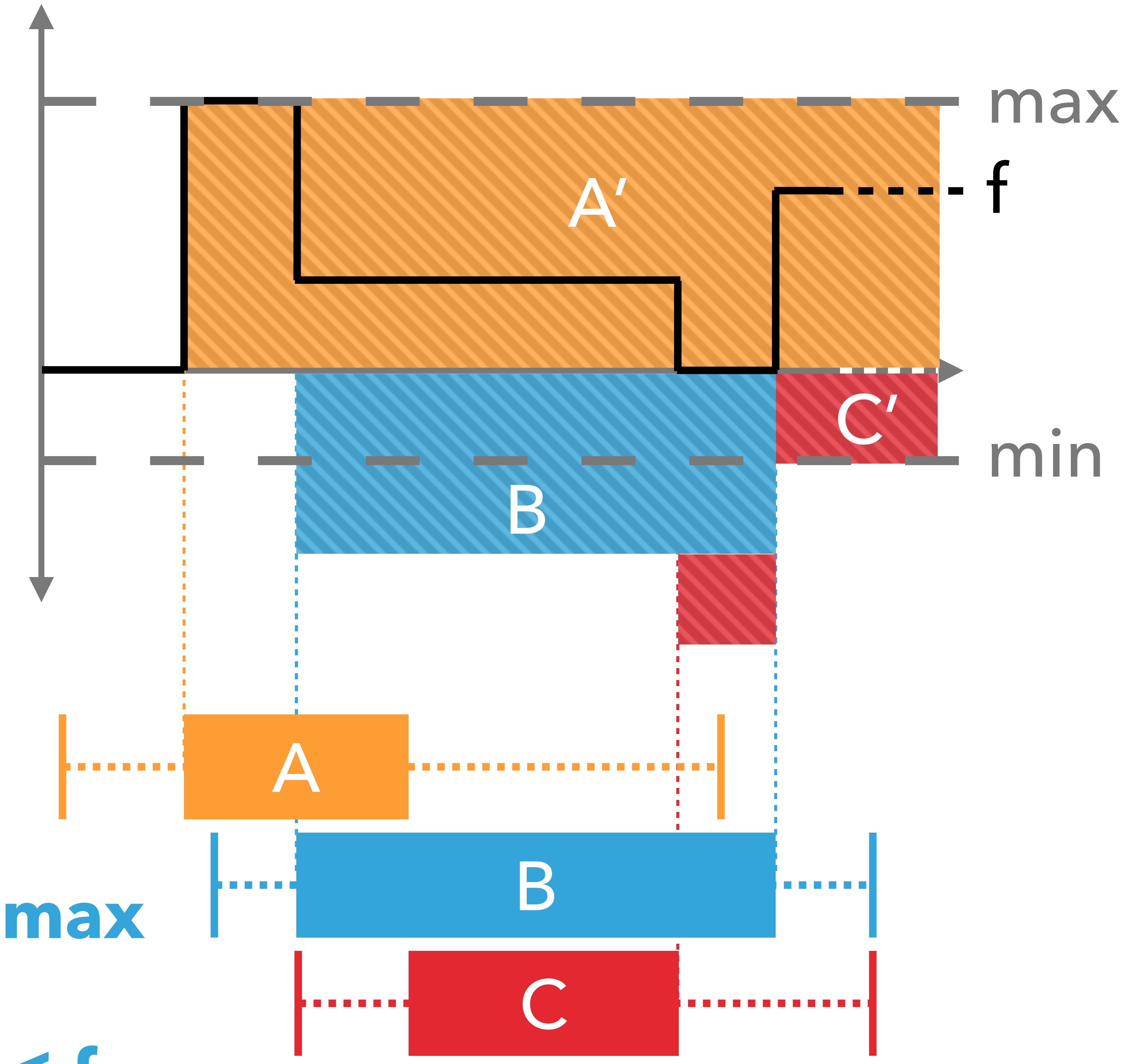

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- **lesserOrEqual(f, max)** = alwaysIn(f, -∞, max) : **f ≤ max**
- **higherOrEqual(f, min)** = alwaysIn(f, min, ∞) : **min ≤ f**
- Collects tasks by **traversing** the AST: {<A′, 3>, <B, -2>, <C′ , -1>}

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- **Heights** can be **variable** too

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- Build **minimum** and **maximum profiles** with contributions of tasks:

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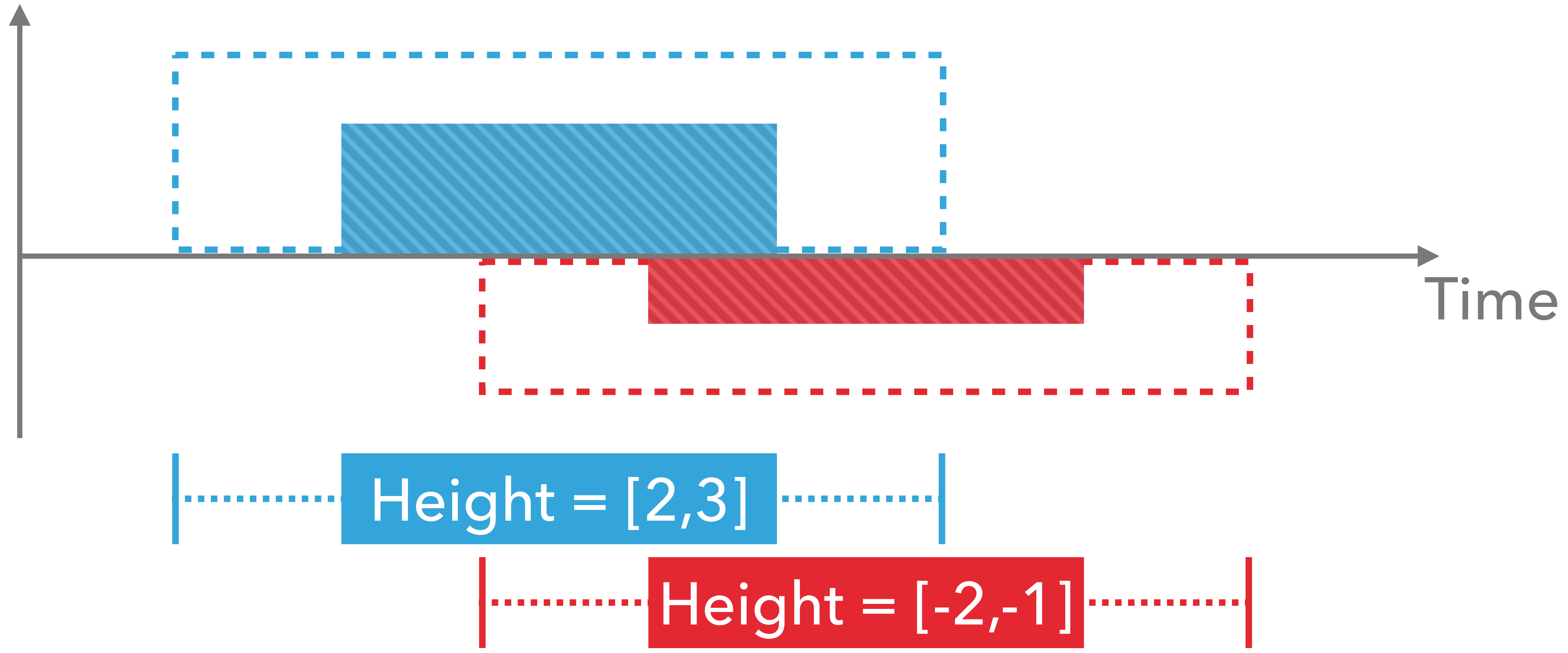

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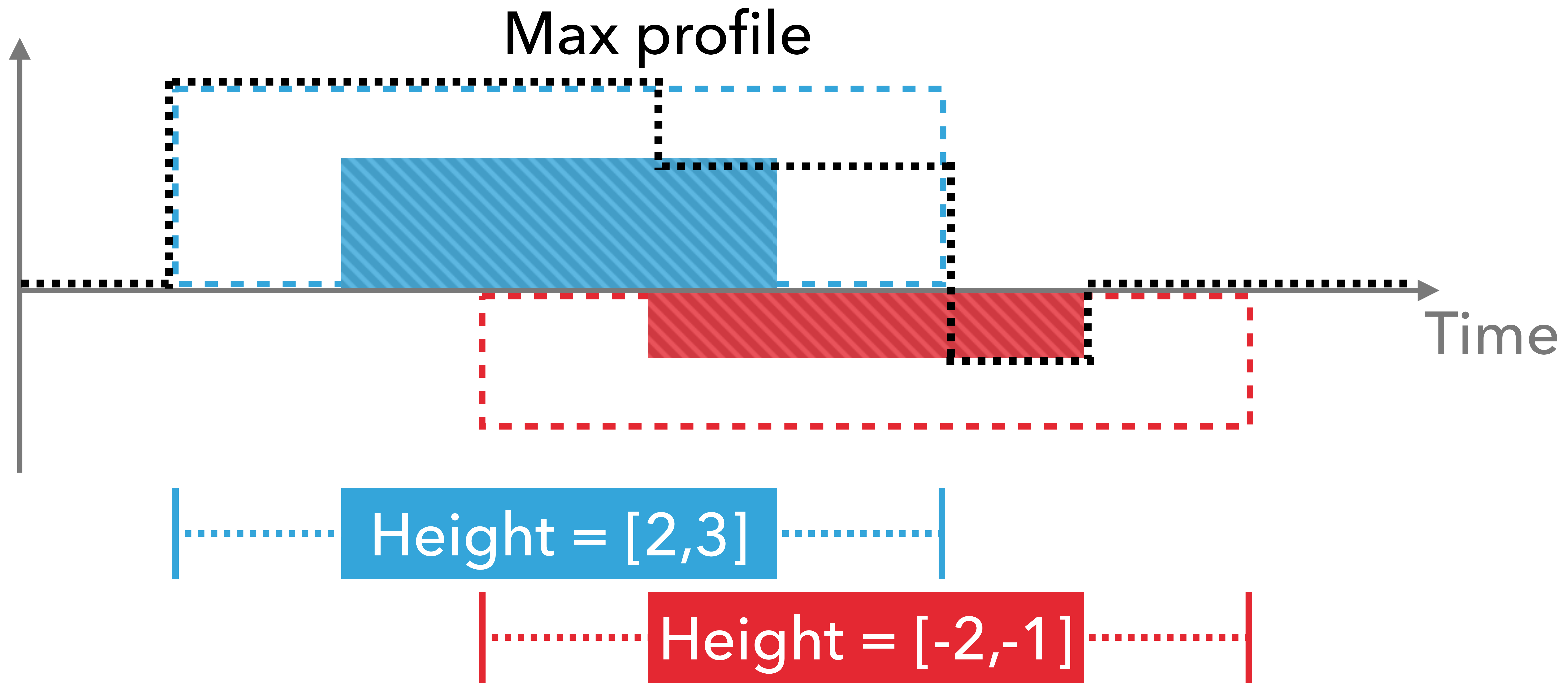

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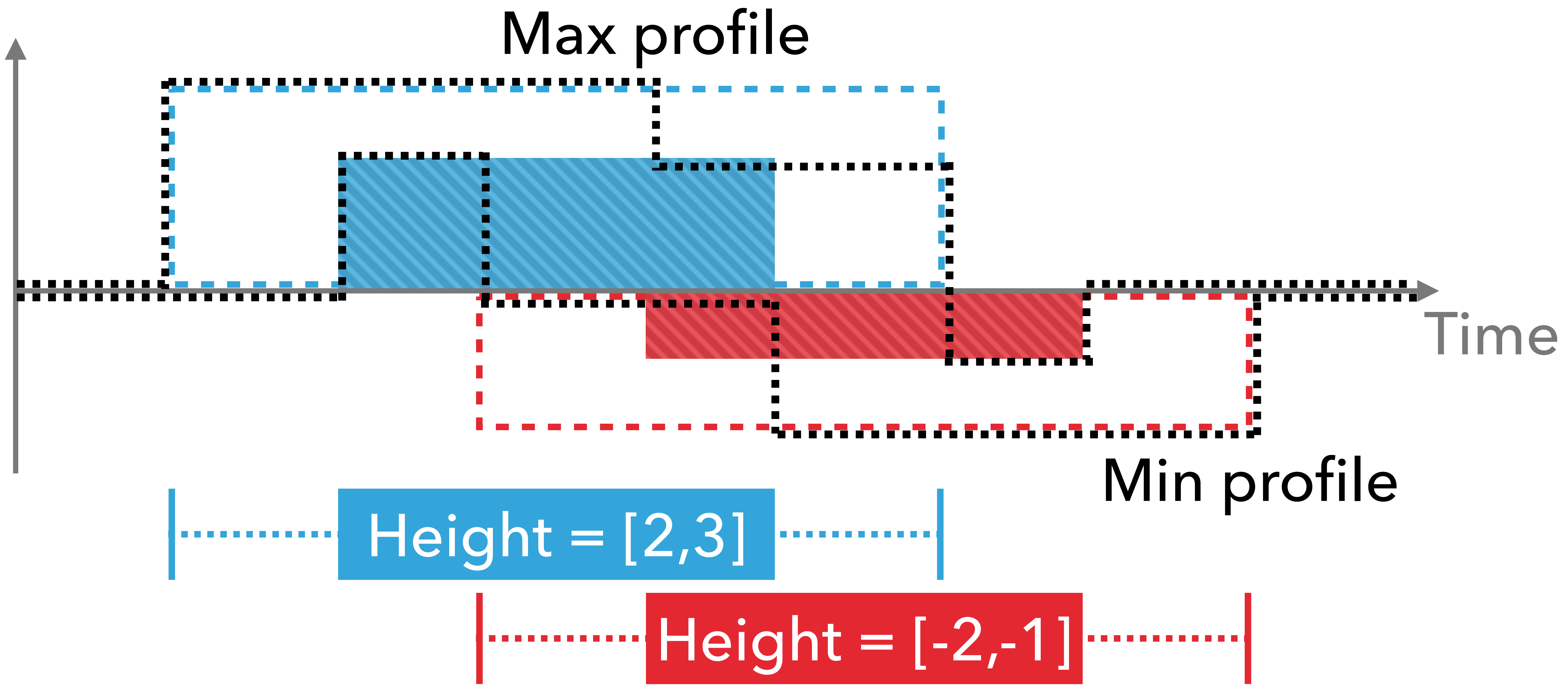

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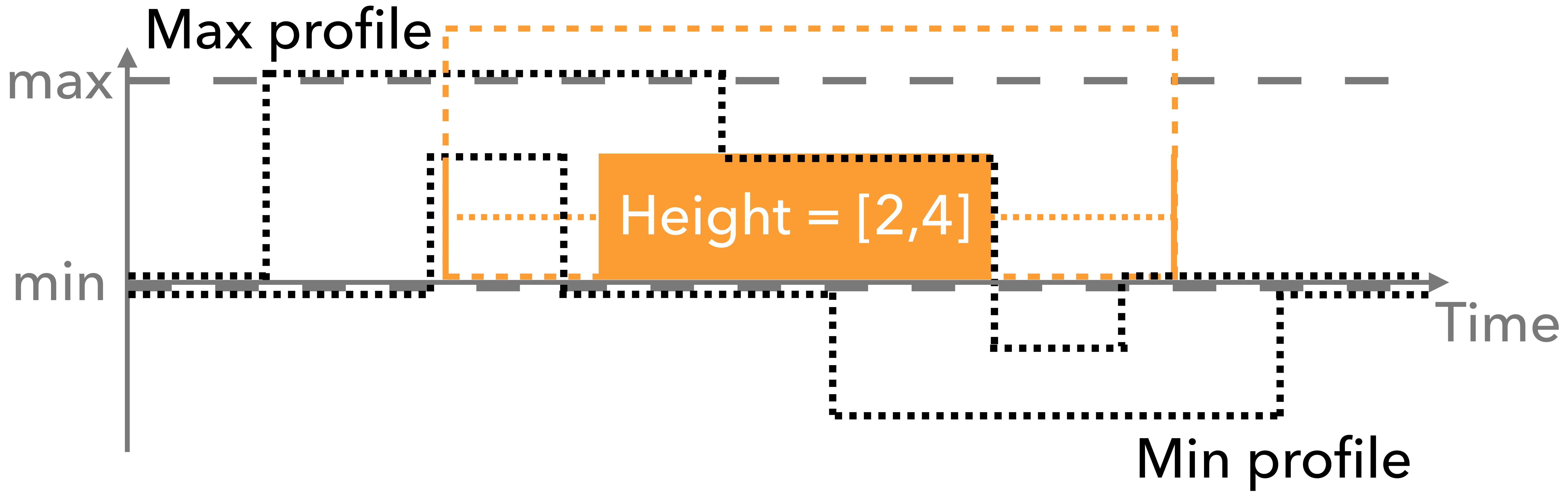
  - Min profile = **max negative** contributions + **min positive** contributions

TIMETABLING



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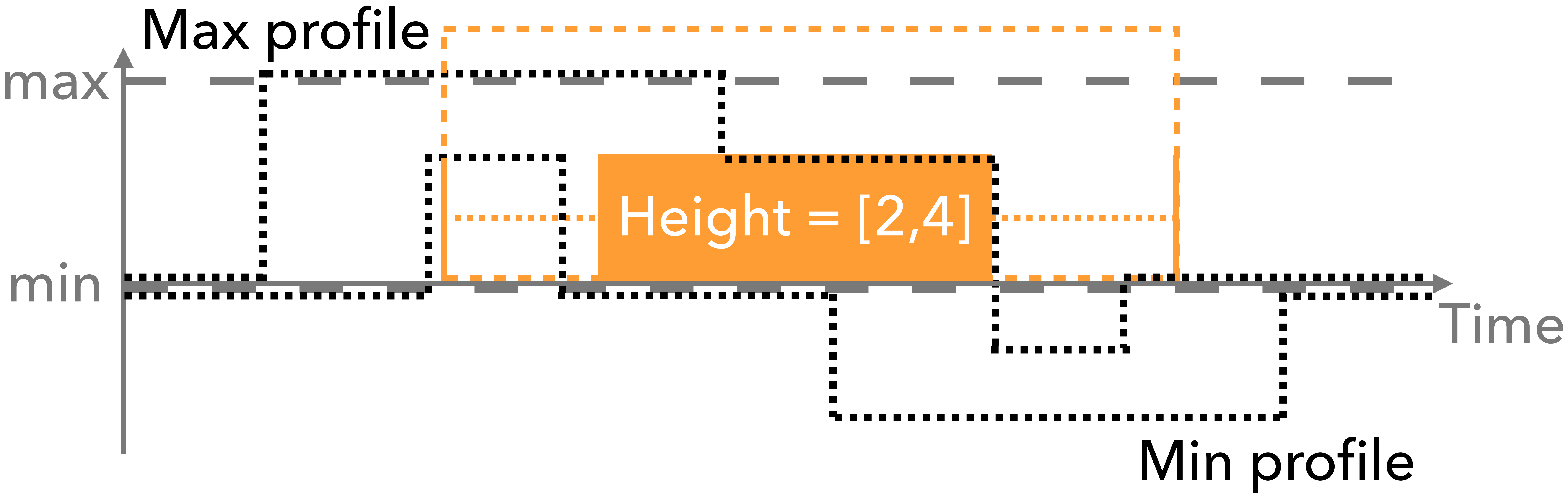

- Compare **each task** during its whole time window with **min** and **max profiles**

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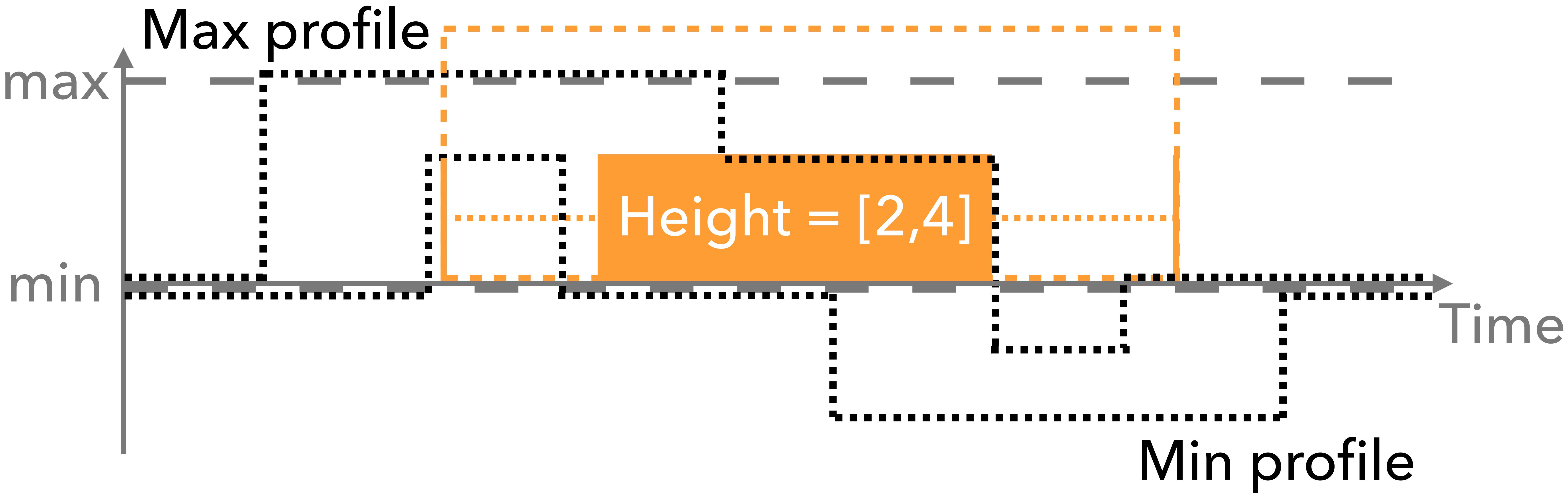

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- 4 propagation rules:

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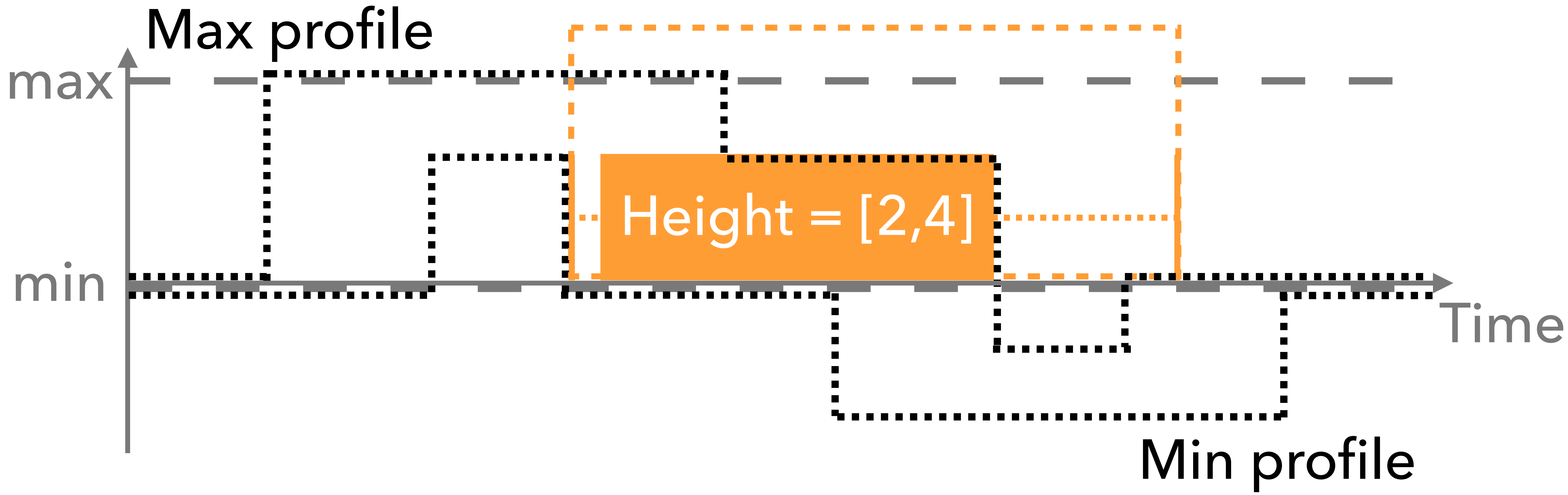

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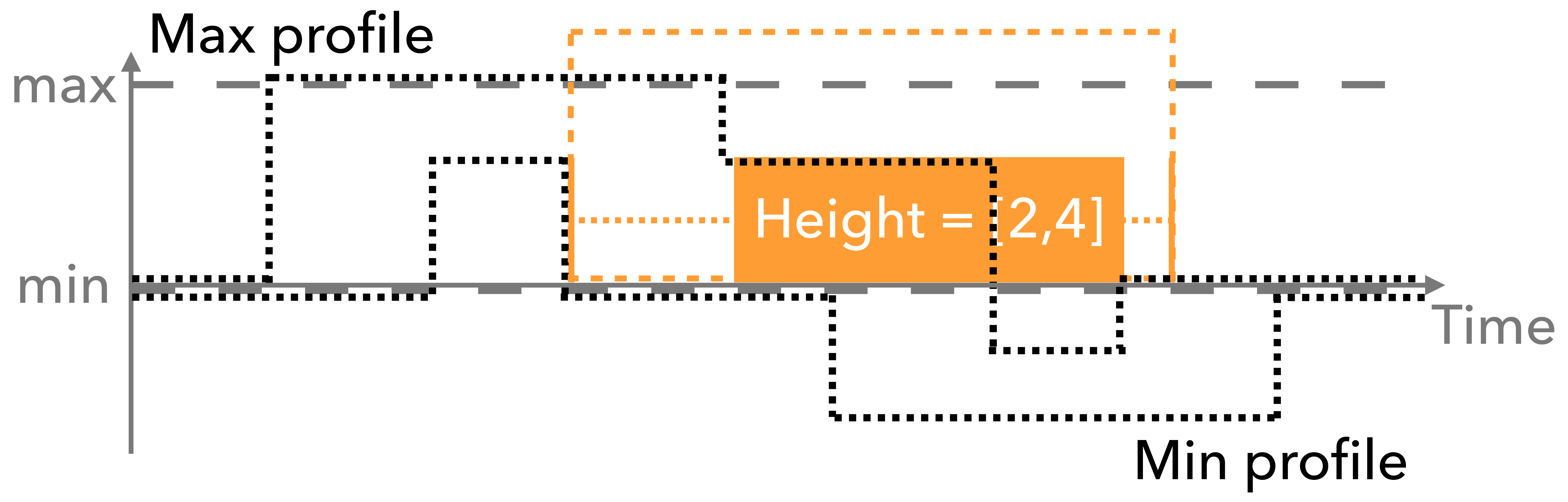

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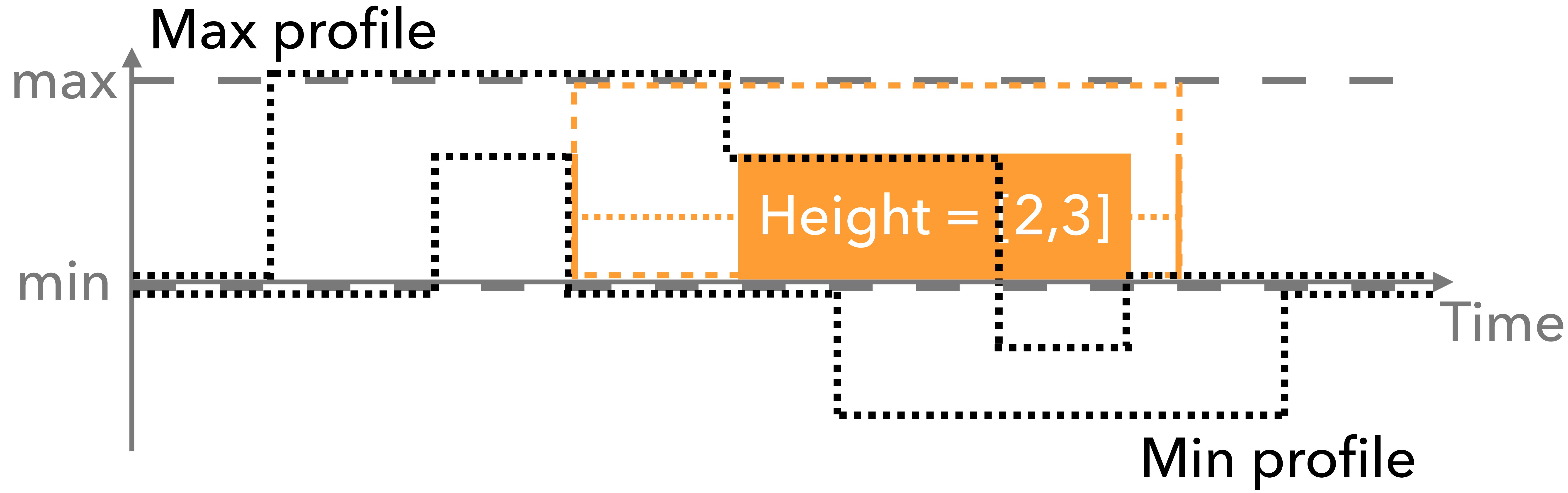

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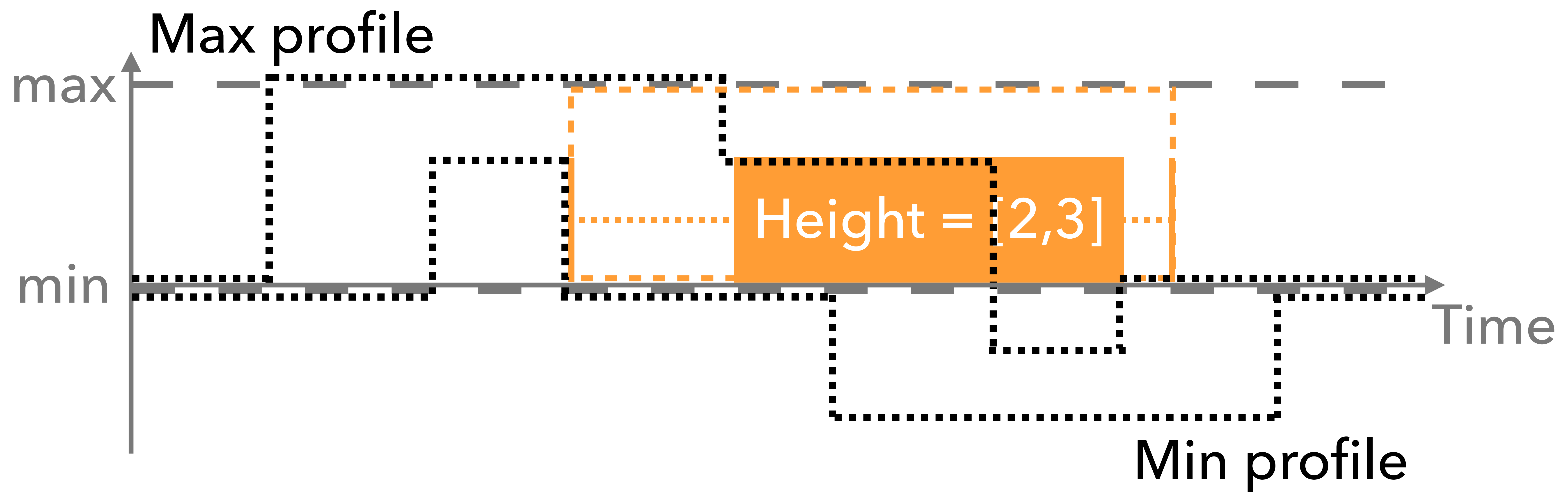

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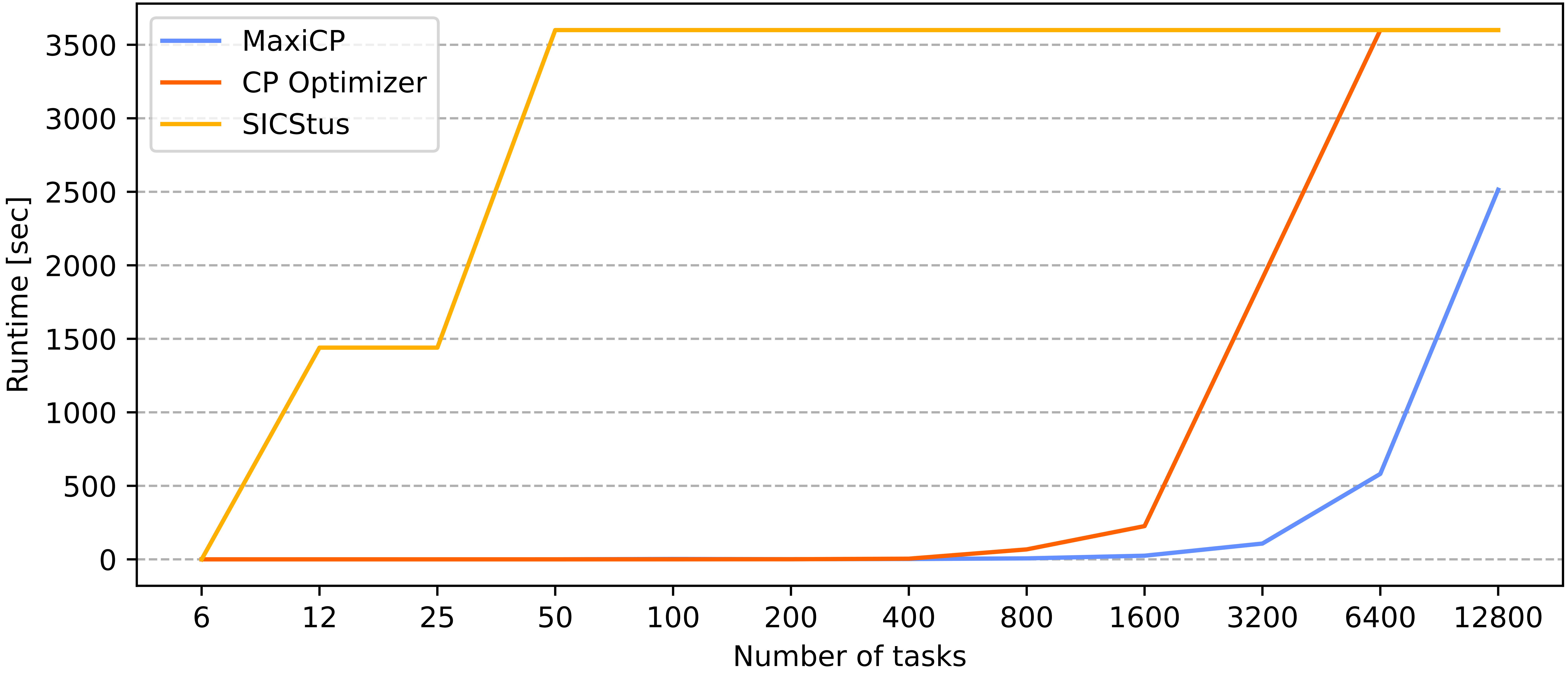
PERFORMANCES



- Maximum Energy Scheduling Problem
- Average time to 1st solution with greedy search, timeout at 3600s

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THANKS FOR YOUR ATTENTION!